11. On the Integro-jump of a Function and Its Fourier Coefficients

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1. Introduction. Suppose that $f(x)$ is periodic with period 2π and Lebesgue integrable in $(-\pi, \pi)$. Let the Fourier series of $f(x)$ be

$$f(x) \sim a_0/2 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

and let

$$\bar{s}_n(x) = \sum_{\nu=1}^{n} (b_\nu \cos \nu x - a_\nu \sin \nu x) \equiv \sum_{\nu=1}^{n} B_\nu(x).$$

We denote by $\bar{\sigma}_n^\alpha(x)$ the n-th Cesàro mean of order α of the sequence $\{\bar{s}_n(x)\}$.

H. C. Chow showed the following

Theorem A.[1] *If there exists a number $L(x)$ such that*

$$\int_0^t \psi(u)\, du = o(t), \quad \int_0^t |\psi(u)|\, du = O(t), \quad \text{as } t \to 0, \tag{1.1}$$

where $\psi(t) = f(x+t) - f(x-t) - L(x)$, *then*

$$\lim_{n\to\infty} \left[\bar{\sigma}_{2n}^\alpha(x) - \bar{\sigma}_n^\alpha(x) \right] = \frac{1}{\pi} \log 2 \cdot L(x), \quad \text{for } \alpha > 0. \tag{1.2}$$

F. C. Hsiang proved also the following

Theorem B.[2] *If the integral*

$$\int_0^t \frac{\psi(u)}{u^{1/\alpha}}\, du \quad (1 > \alpha > 0), \tag{1.3}$$

exists, then

$$\lim_{n\to\infty} \left[\bar{\sigma}_{2n}^1(x) - \bar{\sigma}_n^1(x) \right] = \frac{1}{\pi} \log 2 \cdot L(x). \tag{1.4}$$

Concerning the sequence $\{nB_n(x)\}$, O. Szász[3] proved the following

Theorem C. *Under the assumption of Theorem A, we have*

$$\lim_{n\to\infty} nB_n(x) = -\frac{1}{\pi} L(x) \quad (C, 2). \tag{1.5}$$

Recently Kenzi Yano[4] showed that Theorem C is still valid even if $(C, 2)$ is replaced by $(C, 1+\alpha)$, for every $\alpha > 0$.

It will not be of no interest to replace the conditions of Theorem

1) H. C. Chow: Journ. London Math. Soc., **16**, 23–27 (1941). In this theorem, the case $\alpha = 1$ is O. Szász' theorem (Duke Math. Journ., **4**, 401–407 (1938)).

2) F. C. Hsiang: Bull. Calcutta Math. Soc., **44**, 55–58 (1952).

3) O. Szász: Trans. American Math. Soc., **50** (1942).

4) Kenzi Yano: Nara Joshidai Kiyô (in Jap.), **1** (1951).