35. *Prolongation of the Homeomorphic Mapping*

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(Comm. by K. KUNUGI, M.J.A., March 12, 1955)

1. Mr. G. Choquet enunciated the following theorem[1] "any homeomorphism between two closed bounded subsets of 2-dimensional Euclidean spaces contained in a 3-dimensional Euclidean space can be extended to a homeomorphism of 3-dimensional Euclidean space onto itself".

In this paper we shall give a solution of an analogous theorem in the case of two dimensions i.e. "*any homeomorphism between two closed bounded subsets of* 1-*dimensional Euclidean spaces contained in a* 2-*dimensional Euclidean space can be extended to a homeomorphism of* 2-*dimensional Euclidean space onto itself*".

Let E^i, L^i, F^i $(i=1, 2)$ and $\xi=f(x)$ be respectively the 2-dimensional Euclidean spaces (which are supposed hereafter to be two planes of complex numbers), the real axes of E^i, a closed bounded subsets of L^i, and a given homeomorphism between F^1 and F^2.

Now, we know that a homeomorphism between two Jordan arcs in a Euclidean plane can be extended to that of the whole plane by using the correspondences between the boundaries in conformal mappings[2] and the correspondence between the corresponding radii of two unit circles. Therefore, in order to prove the theorem, it is sufficient to show that we can construct a Jordan arc J which has the following properties: J is homeomorphic to the closed interval $[a, b]$, where a and b are the two end-points of F^1, and this homeomorphism between J and $[a, b]$ is an extension of the given homeomorphism $\xi=f(x)$.

2. In the first place, we assume that F^i is totally disconnected.

Since the derived set $(F^1)'$ is closed, $L^1-(F^1)'$ is open in L^1, therefore $L^1-(F^1)'$ is a sum set of an at most enumerable number of disjoint open intervals. As the number of those intervals, whose lengths are longer than a positive number $\rho>0$, is finite, we can enumerate all of the bounded disjoint open intervals in the order of their lengths. We denote them by

$$(x^*_{n,1}, x^*_{n,2}),\quad x^*_{n,1}(\in(F^1)')<x^*_{n,2}(\in(F^1)'),\quad x^*_{n,2}-x^*_{n,1}\geqq x^*_{n+1,2}-x^*_{n+1,1}$$

(where $x^*_{n,2}>x^*_{n+1,2}$ in the case of equality), $n=1, 2, 3, \dots$. Since $(x^*_{n,1}, x^*_{n,2})\cap F^1$ can have only isolated points[3] of F^1, the order type

1) Comptes Rendus de l'Académie des Sciences de Paris, **219**, 542 (1944).

2) Cf. Hurwitz-Courant: Funktionentheorie, 400-405 (1929).

3) Inversely, it is clear that any isolated point F^1 belongs to one of the sets $(x^*_{n,1}, x^*_{n,2})\cap F^1$, $n=1, 2, 3, \dots$