34. *Note on an Extension of Multiplication of Distributions*

By Tadashige ISHIHARA
Department of Mathematics, Osaka University
(Comm. by K. KUNUGI, M.J.A., March 12, 1955)

Multiplication of distributions are considered by L. Schwartz (1) in his text-book in case only when one of the distributions is a non-function at each step of multiplication. Indeed, according to the ordinary definition we can not consider other sort of multiplication.

Meanwhile in some applied branches (for example in the calculations of S-matrix by the power series expansions of coupling constant (J. Schwinger (2), F. J. Dyson (3), etc.)), we meet usually rather extraordinary multiple several numbers of whose multiplicand are non-function distributions. So it will be desired to examine the possibility of extension of its definition to the case in which more than two non-functions can occur. (We shall need such examinations also in the case when we want to examine whether or not we are able to consider the non-linear equation whose solution is a non-function distribution.)

Recently L. Schwartz (4) has pointed out the impossibility of the associative multiplication including δ and the derivative operation from purely algebraic consideration. In this paper we study the extended multiplication mainly from the topological consideration. That is to say, if the multiplying operation $T \to QT$ by a fixed distribution Q, is defined by the contragradient mappings $\varphi \to \varphi Q$ for $\varphi \in \mathfrak{D}$, then the structure of the space $\mathfrak{Q} = \{\varphi Q \mid \varphi \in \mathfrak{D}\}$ determines the nature of the multiplication. So if we require some conditions for the topology of the space $\mathfrak{Q}$ (whose algebraic structure is assumed to be the same as in the space $\mathfrak{D}'$), we can determine the range of the multiplicands and the multiples independently from the other algebraic requirements such as the law of the derivation or of the association except the linearity of the multiplication which is always assured by this sort of definition.

The main result of this paper studied along this line is the following: Considering two multiplicands, if we impose a condition (C) upon the extended multiplication, then we can consider at most a multiple T such that either T is essentially an ordinary multiple of Q and of $\alpha \in \mathfrak{D}$ or T is a limit in $\mathfrak{D}'$ of ordinary multiple $\alpha_\lambda Q$.

Concerning the terminologies used in this paper, see for example L. Schwartz (1), N. Bourbaki (5), and J. Dieudonné (6).

1. For a fixed distribution $Q (\in \mathfrak{D}')$ we consider the vector space