33. *Remarks on the Jordan-Hölder-Schreier Theorem**)

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The Jordan-Hölder-Schreier theorem, or shortly the J-H-S theorem, in lattices has been considered as the formulation of the J-H-S theorem for algebraic systems. But, A. W. Goldie has proved in his paper [3] the usual theorem on lengths of chains in modular lattices, using the Jordan-Hölder theorem for algebraic systems. In this note, the relations between these theorems will be more cleared. First, we shall show the J-H-S theorem for algebraic systems (§ 1). Next, considering a lattice L as the algebraic system with the composition $\cup$ only, we shall introduce a theorem for normal chains of L as the special case of the above theorem. And this theorem will be shown to be the usual J-H-S theorem in the lattice L (§ 2).

§ 1. **Algebraic Systems.** In this note we put the following conditions on the algebraic system A to keep out the complication:

0. All compositions are binary and single-valued, moreover any two elements may be composable by any composition.

I. A has a null-element e.

We denote by $\theta(B)$, $\varphi(B),\dots$ the congruences on a subsystem B of A. Moreover we denote by Θ the set of all congruences on all subsystems of A, i.e. $\Theta=\{\theta(B):B\subset A\}$.

Two congruences $\theta(B)$ and $\varphi(C)$ are called *weakly permutable* if and only if

$$(S(\theta(B\frown C))\,|\,\varphi(B\frown C))=(S(\varphi(B\frown C))\,|\,\theta(B\frown C)).$$

Moreover a congruence $\omega(B\frown C)$ is called a *quasi-join* of $\theta(B)$ and $\varphi(C)$, if and only if

i) $\omega(B\frown C)\geqq\theta(B\frown C)\smile\varphi(B\frown C)$ and

ii) $S(\omega(B\frown C))=S(\theta(B\frown C)\smile\varphi(B\frown C))$.

A subset Φ of Θ is called a *normal family*, when Φ has the following conditions:

i) Any two congruences in Φ are weakly permutable.

ii) For any congruences $\theta(B)$, $\varphi(C)$ in Φ, there exists a quasi-join $\omega(B\frown C)\in\Theta$ such that $[\omega(B\frown C)\,|\,\theta(B)]$, $[\omega(B\frown C)\,|\,\varphi(C)]\in\Phi$. Such a quasi-join $\omega(B\frown C)$ is called a *normal quasi-join*.

A normal chain

$$M=A_0\supset S(\theta_0(A_0))=A_1\supset\cdots\supset S(\theta_{r-1}(A_{r-1}))=A_r=N$$

*) In this note, we shall use the theorems, the terms and the notations in [1] and [2], without the explanations.