32. *Note on the Isomorphism Problem for Free Algebraic Systems*

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Let $V=\{\alpha_1, \alpha_2, \dots\}$ be the system of single-valued compositions, and A the family of composition-identities with respect to V, and let $E=\{a_1, a_2, \dots\}$ be the free generator system. Then it is easily verified that the free A-algebraic system $A^V(E)$, or shortly $A(E)$, can be defined. Let $F=\{b_1, b_2, \dots\}$ be another free generator system. If the cardinal numbers of E and F are equal, then it is clear that $A(E)\cong A(F)$.*)

In this note we shall show that, under some conditions of $A(E)$, $A(E)\cong A(F)$ if and only if the cardinal numbers of E and F are equal, i.e. we shall give the solution of the isomorphism problem for the free A-algebraic system satisfying such conditions. And the isomorphism problems of free groups, free lattices, and others can be easily solved as the special cases of our results.

Theorem I. *Let $A(E)$ be a free A-algebraic system satisfying the following two conditions:*

1) *the composition-identity $x=y$ is not derived from A,*

2) *the cardinal number of E is infinite.*

Then $A(E)\cong A(F)$ if and only if the cardinal numbers of E and F are equal.

Proof. "If"-part of this theorem is immediate. Hence we shall prove "only if"-part.

Let $E=\{a_1, a_2, \dots\}$ and $F=\{b_1, b_2, \dots\}$. Now suppose that $\overline{E}>\overline{F}$**) in spite of $A(E)\cong A(F)$. First we can suppose $A(E)=A(F)$ instead of $A(E)\cong A(F)$, without loss of generality. Hence $b_1, b_2, \dots$ are represented by finite compositions of finite elements in E respectively, i.e.

$$b_1=f_1(E),\ b_2=f_2(E), \dots .$$

Let E' be the set of all the elements in E which appear in some $f_i(E)$. Then the cardinal number of E' is smaller than $\overline{E}$. Hence there exists an element $a_j\in E$ such that $a_j\notin E'$. And a_j is also represented by finite compositions of finite elements in F, i.e. $a_j=\varphi(F)$. Putting $f_1(E), f_2(E), \dots$ in places of $b_1, b_2, \dots$ respectively, we get $a_j=\psi(E)$. Taking off unnecessary elements from E in this identity, we get $a_j=\psi(E'')$, where E'' is a finite set contained in E'.

*) K. Shoda: Allgemeine Algebra, Osaka Math. J., **1** (1949).

**) $\overline{E}$ denotes the cardinal number of E.