26. *Convergence of Fourier Series*

By Masako SATÔ
Mathematical Institute, Tokyo Metropolitan University, Tokyo
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1. G. H. Hardy and J. E. Littlewood [1] proved the following theorem concerning the convergence of Fourier series at a point.

Theorem HL. *If*

$$\int_0^t \left|\varphi_x(u)\right| du = o\left(t \Big/ \log \frac{1}{t}\right) \qquad (t \to 0) \tag{1}$$

and

$$\int_0^t |\, d(u^{\Delta}\varphi_x(u))\,| = O(t) \qquad (\Delta > 1), \tag{2}$$

then the Fourier series of $f(t)$ converges at $t=x$.

Recently G. Sunouchi [2] proved the following

Theorem S. *If (1) holds and*

$$\lim_{k\to\infty} \limsup_{h\to 0} \int_{(hk)^{1/\Delta}}^{\eta} \left| \frac{\varphi_x(t)-\varphi_x(t+h)}{t} \right| dt = 0 \qquad (\Delta > 1,\ \eta > 0), \tag{3}$$

then the Fourier series of $f(t)$ converges at $t=x$.

The object of this paper is to prove a convergence theorem similar to Theorem S, replaced the first condition by the weaker in order and the second condition by the stronger. More precisely we prove the following

Theorem 1. *Let $0<\alpha<1$. If*

$$\varphi_x(t)-\varphi_x(t') = o\left(1 \Big/ \left(\log \frac{1}{|t-t'|}\right)^{\alpha}\right) \qquad (t, t' \to 0) \tag{4}$$

and

$$\lim_{n\to\infty} \int_{\pi e^{(\log n)^{\alpha}}/n}^{\eta} \left| \frac{\varphi_x(t)-\varphi_x(t+\pi/n)}{t} \right| dt = 0 \qquad (\eta > 0), \tag{5}$$

then the Fourier series of $f(t)$ converges at $t=x$.

As S. Izumi and G. Sunouchi [3] have proved, in the case $\alpha \geqq 1$ the Fourier series of $f(t)$ converges uniformly at $t=x$ without the second condition.

Theorem 2. *Let $\alpha>0$. If*

$$\varphi_x(t)-\varphi_x(t') = o\left(1 \Big/ \left(\log\log \frac{1}{|t-t'|}\right)^{\alpha}\right) \qquad (t, t' \to 0) \tag{6}$$

and

$$\lim_{n\to\infty} \int_{\pi e^{(\log\log n)^{\alpha}}/n}^{\eta} \left| \frac{\varphi_x(t)-\varphi_x(t+\pi/n)}{t} \right| dt = 0 \qquad (\eta > 0) \tag{7}$$

then the Fourier series of $f(t)$ converges at $t=x$.