47. On the Riesz Logarithmic Summability of the Conjugate Derived Fourier Series. II[1]

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5. Proof of Theorem 2. We shall consider the integral

$$I_1=\frac{1}{(\log\omega)^{\alpha+1}}\int_0^\pi g_\alpha(t)\frac{1-\cos\omega t}{t}dt,\quad (\alpha\geqq 0),$$
$$=\frac{1}{(\log\omega)^{\alpha+1}}\left\{\int_0^{\pi/\omega}+\int_{\pi/\omega}^\pi\right\}=I_{1,1}+I_{1,2},$$

say. Integrating by parts, we have

$$I_{1,1}=\frac{1}{(\log\omega)^{\alpha+1}}\left[g_\alpha^1(t)\frac{1-\cos\omega t}{t}\right]_0^{\pi/\omega}$$
$$-\frac{1}{(\log\omega)^{\alpha+1}}\int_0^{\pi/\omega}g_\alpha^1(t)\frac{t\omega\sin\omega t-(1-\cos\omega t)}{t^2}dt$$
$$=o\left[\frac{1}{(\log\omega)^{\alpha+1}}(\log\omega)^\alpha\right]+o\left[\frac{\omega^2}{(\log\omega)^{\alpha+1}}\int_0^{\pi/\omega}t\left(\log\frac{1}{t}\right)^\alpha dt\right]$$
$$=o(1/\log\omega)=o(1),$$

since $g_\alpha^1(t)=o[t(\log 1/t)^\alpha]$ by the assumption of Theorem 2. Also

$$I_{1,2}=\frac{1}{(\log\omega)^{\alpha+1}}\int_{\pi/\omega}^\pi\frac{g_\alpha(t)}{t}dt-\frac{1}{(\log\omega)^{\alpha+1}}\int_{\pi/\omega}^\pi\frac{g(t)}{t}\cos\omega t\,dt$$
$$=I_{1,2,1}-I_{1,2,2},$$

say, where

$$I_{1,2,1}=\frac{1}{(\log\omega)^{\alpha+1}}\left[\frac{g_\alpha^1(t)}{t}\right]_{\pi/\omega}^\pi+\frac{1}{(\log\omega)^{\alpha+1}}\int_{\pi/\omega}^\pi g_\alpha^1(t)\frac{1}{t^2}dt=o(1)$$

and

$$2(\log\omega)^{\alpha+1}I_{1,2,2}=2\int_{\pi/\omega}^\pi g_\alpha(t)\frac{\cos\omega t}{t}dt$$
$$=\int_{\pi/\omega}^{2\pi/\omega}g_\alpha(t)\frac{\cos\omega t}{t}dt+\int_\pi^{\pi+\pi/\omega}g_\alpha(t)\frac{\cos\omega t}{t}dt$$
$$+\int_{\pi/\omega}^\pi\left\{\frac{g_\alpha(t)}{t}-\frac{g_\alpha(t+\pi/\omega)}{t+\pi/\omega}\right\}\cos\omega t\,dt.$$

The first term of the above expression is $o[(\log\omega)^{\alpha+1}]$, as in the estimation of $I_{1,1}$ and the second term is $o(1)$, as easily may be seen. On the other hand, the third term becomes

$$\int_{\pi/\omega}^\pi\frac{g_\alpha(t)-g(t+\pi/\omega)}{t}\cos\omega t\,dt$$

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1) Continued from p. 125. References are cited on p. 125.