61. On the Structure of Semigroups Containing Minimal Left Ideals and Minimal Right Ideals

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For the semigroup S containing minimal left ideals and minimal right ideals, we can define the kernel D of S as the product of any one minimal left ideal L and minimal right ideal R. It should be noted that D does not depend on the choice of L and R. The previous works in which the structure of the kernel is treated have already been given in [1], [2], and [3]. Recently the present author has shown the following:

Let D_{kl}, $k=1,2,\dots,p;l=1,2,\dots,q$, where p is the number of minimal right ideals and q one of minimal left ideals in the kernel D, be the groups which compose D and have no element in common, then every minimal left ideal L and every minimal right ideal R can be represented in the form

$$L_l=\sum_{k=1}^{p}D_{kl}$$
$$R_k=\sum_{l=1}^{q}D_{kl}, \tag{1}$$

and among these groups,

$$D_{jl}=D_{jm}D_{kl} \tag{2}$$

holds true [4].

The purpose of the present paper is to show that the semigroup S containing minimal left ideals and minimal right ideals has the similar structure as that of the kernel under the condition weaker than the cancellation law.

In the present paper we use certain of the ideas, notations, and results given in the previous paper [4] without explanation.

We have already seen that d is an element of a minimal left ideal L_l, then $d\,\varepsilon\,D_{kl}$ for a certain k, and $Sd=L_l$. Let S_{kj} be the set of elements $s\,\varepsilon\,S$ such that $sd\,\varepsilon\,D_{jl}$, then $S=\sum_{j=1}^{p}S_{kj}$ and $S_{kl}\cap S_{kj}=\phi$ since $D_{il}\cap D_{jl}=\phi$ for $i\neq j$. We shall call this decomposition of S the s-decomposition.

For R_j, the minimal right ideal including D_{jl}, $R_jD_{kl}=D_{jl}$ by (2), therefore,

$$S_{kj}\supset R_j \qquad \text{for all } k. \tag{3}$$

Let d' be any element of D_{kl}, then for $s_{kj}\,\varepsilon\,S_{kj}$, $s_{kj}d'\,\varepsilon\,s_{kj}(dE)\,\varepsilon\,D_{jl}E=D_{jl}$ where E is any one of groups composing L_l, and so we