# 99. Groups of Isometries of Pseudo-Hermitian Spaces. II

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In the previous paper [4] we have given Theorem 4, in which the assumption $n \neq 3$, $n>1$ must be added. The notation and assumptions being as in the previous, we can state the following theorem including the case $n=3$.

THEOREM 4. *Let $G/H$ be a homogeneous pseudo-Hermitian space of dimension $2n$ and $\dim G=n^2+2n-1$ $(n>1)$. If $n \neq 3$, $G/H$ is flat and homeomorphic to $E^{2n}$ and the group $G$ is isomorphic to $S\mathfrak{M}_H(n)$. If $n=3$, $G/H$ is flat or of positive constant curvature.*

*In case $n=3$ and $G/H$ is flat, the conclusion is the same as in the general case. In case $n=3$ and $G/H$ is of positive constant curvature, $G/H$ is homeomorphic to a sphere $S^6$ of dimension 6 and the group $G$ is isomorphic to a compact exceptional simple group of type $(G)$.*

PROOF. Since $H$ is isomorphic to $SU(n)$, there exists in the Lie algebra $\mathfrak{g}$ a subspace $\mathfrak{m}$ such that

$$\mathfrak{g}=\mathfrak{m}+\mathfrak{h} \qquad \text{(direct sum as vector space)},$$
$$[\mathfrak{h}, \mathfrak{m}] \subset \mathfrak{m},$$

where $\mathfrak{h}$ is the subalgebra of $\mathfrak{g}$ corresponding to the subgroup $H$ and $[\mathfrak{h}, \mathfrak{m}]$ denotes the subspace spanned by all elements of the form $[U, X]$, $U \in \mathfrak{h}$, $X \in \mathfrak{m}$.

First, we have easily

$$[\mathfrak{h}, [\mathfrak{m}, \mathfrak{m}]] \subset [\mathfrak{m}, \mathfrak{m}],$$

where $[\mathfrak{m}, \mathfrak{m}]$ denotes the subspace spanned by all elements of the form $[X, Y]$, $X, Y \in \mathfrak{m}$. Since $\mathfrak{h}$ is simple, one of the following four cases occurs:

$$[\mathfrak{m}, \mathfrak{m}]=\{0\}, \quad [\mathfrak{m}, \mathfrak{m}]=\mathfrak{m}, \quad [\mathfrak{m}, \mathfrak{m}]=\mathfrak{h}, \quad [\mathfrak{m}, \mathfrak{m}]=\mathfrak{g}.$$

The first three cases have been discussed in the previous paper. When the last case occurs, it is easily seen that $\mathfrak{g}$ is simple and $\dim \mathfrak{g}=n^2+2n-1$. Looking over the table of simple Lie algebras due to É. Cartan [2, p. 49], we see that, if there exists such a simple Lie algebra $\mathfrak{g}$, $n$ must be 3 and $\mathfrak{g}$ the exceptional simple Lie algebra $\mathfrak{g}_{14}$ of type $(G)$ which is of rank 2 and of dimension 14. Thus Theorem 4 is proved for $n \neq 3$.

To prove Theorem 4 for $n=3$, assume that $\mathfrak{g}$ has the structure of the exceptional simple Lie algebra of type $(G)$. It has been proved by É. Cartan [3, pp. 292-298] that there exist exactly two