98. *On Factor Set of the Third Obstruction*

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The object of the present note[1] is to give the third obstruction theorem for mappings of a geometric complex K into a topological space Y such that

$$\pi_i(Y)=0 \quad \text{for } 0\leq i<n,\ n<i<q, \text{ and } q<i<r<2q-1,$$

along the line of Eilenberg-MacLane.[2]

For such a space Y we described previously[3] the cohomology class $\boldsymbol{k}_{n.q}^{r+1}$ of $H^{r+1}(\pi_n, \pi_q, \boldsymbol{k}_n^{q+1}; \pi_r)$[4] as a topological invariant if we pay no heed to the identification of the complexes $K(\pi_n, n, \pi_q, q, k)$, where $\boldsymbol{k}_n^{q+1}=\boldsymbol{k}_n^{q+1}(Y)$ is the Eilenberg-MacLane invariant of the space Y.

In this paper we shall introduce new operators $\boldsymbol{y}_T$ and $\boldsymbol{y}_\tau$. And by making use of $\boldsymbol{k}_{n.q}^{r+1}{}_T$, $\boldsymbol{k}_{n.q}^{r+1}{}_\tau$, we shall describe a factor set of the third obstruction of a map.

1. As a preliminary to the definition of the basic operators, we shall consider first certain maps.

We wish to classify simplicial maps T of a C.S.S. complex K in $K(\Pi, n, \Pi', q, k)$. Such a map determines a cocycle $x_n=T^{\#}b_n \in Z^n(K; \Pi)$ and a cochain $x_q=T^{\#}b_q \in C^q(K; \Pi')$, where b_n is the basic cocycle in $Z^n(\Pi, n, \Pi', q, k; \Pi)\cong Z^n(\Pi, n; \Pi)$ and b_q is the basic cochain in $C^q(\Pi, n, \Pi', q, k; \Pi')$ defined by setting

$$b_n(\phi, \psi)=\phi(\varepsilon_n), \qquad b_q(\phi, \psi)=\psi(\varepsilon_q).$$

Lemma 1. Given the complex $K(\Pi, n, \Pi', q, k)$ *and the C.S.S. complex* K, *the rule* $T\rightarrow(x_n, x_q)$ *establishes a one to one correspondence between simplicial maps and pairs* (x_n, x_q) *satisfying the conditions:*

$$x_n \in Z^n(K; \Pi),\ x_q \in C^q(K; \Pi'),\ kT(x_n)+\delta x_q=0.$$

The map T corresponding in this fashion to the pair (x_n, x_q) will be denoted by $T(x_n, x_q)$. Then $T(x_n, x_q)$ is characterized as

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1) Full details will appear in the Journal of the Institute of Polytechnics, Osaka City University.

2) S. Eilenberg and S. MacLane: On the groups $H(\Pi, n)$, III, Ann. Math., **60**, 513–557 (1954). Present note makes full use of the results and terminology of this paper.

3) K. Mizuno: On the minimal complexes, Jour. Inst. Polytech., Osaka City Univ., **5**, 41–51 (1954).

4) For the sake of brevity, we write in the following $\pi_n=\pi_n(Y)$, $\pi_q=\pi_q(Y)$, and $\pi_r=\pi_r(Y)$.