# 142. *Uniform Convergence of Fourier Series.* V

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1. R. Salem [1] proved the following theorem concerning uniform convergence of Fourier series.

**Theorem 1.** (i) *If* $f(x)$ *is continuous and*

$$(1)\qquad \frac{1}{h}\int_0^h (f(x+t)-f(x-t))dt=o\left(1\Big/\log\frac{1}{h}\right)\quad \text{as } h\to 0$$

*uniformly for all* $x$, *then the Fourier series of* $f(x)$ *converges uniformly everywhere.*

(ii) *If* $f(x)$ *is continuous in* $[a, b]$ *and the condition* (1) *is satisfied uniformly for* $x$ *in* $[a, b]$, *then the Fourier series of* $f(x)$ *converges uniformly in* $[a+\eta, b-\eta]$.

(iii) *If* (1) *holds uniformly in* $(a, b)$, *then the Fourier series of* $f(x)$ *converges almost everywhere in* $(a, b)$.

On the other hand, S. Izumi and G. Sunouchi [2] proved the following theorem concerning uniform convergence of Fourier series at a point.

**Theorem 2.** If

$$(2)\qquad f(t)-f(t')=o\left(1\Big/\log\frac{1}{|t-t'|}\right)\quad \text{as } t, t'\to x,$$

then the Fourier series of $f(t)$ converges uniformly at $t=x$.

S. Izumi-G. Sunouchi [2] and the author [3] proved theorems concerning uniform convergence of Fourier series at a point, under the conditions weaker than (2), with additional condition on the order of the Fourier coefficients of $f(x)$.

The object of this paper is to prove Theorem 1 by the method of R. Salem used in [2] and [3]. Further we prove theorems in [2] and [3], replaced uniform convergence at a point by that in an interval and their continuity conditions by those of type (1). We prove also similar theorems concerning ordinary convergence.

Finally we prove an improvement of another theorem of R. Salem [1], which gives the majorant of the partial sum of Fourier series.

2. We shall prove first Theorem 1, (i). Let $s_n(x)$ be the $n$th partial sum of the Fourier series of $f(x)$. Then it is sufficient to prove that $s_n(x)-f(x)=o(1)$, unif. for odd $n$. We put

$$s_n(x)-f(x)=\frac{1}{\pi}\int_0^\pi \varphi_x(t)\frac{\sin nt}{t}dt+o(1)=\frac{1}{\pi}\left[\int_0^{\pi/n}+\int_{\pi/n}^{\pi}\right]+o(1)$$
$$=I+J+o(1).$$