167. Vector-space Valued Functions on Semi-groups. III

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In this Note, we shall define the Maak function and prove the existence for almost periodic functions. We shall use the terminologies in my Note [5], [7]. The method is due to W. Maak [3].

V. Fundamental theorem on almost periodic function

Let $f(x)$ be an almost periodic function on a semi-group G with unit into a locally convex vector space E. For any nbd U of E, we have a minimal decomposition of G. The following propositions are clear.

Proposition 5.1. For any nbd U and an almost periodic function, G has a minimal decomposition.

Proposition 5.2. Let $\{A_i\}$ $i=1, 2, \dots, n$ be a minimal decomposition of G for any almost periodic function, then for a, b of G,

$$A_i \frown aGb \neq 0 \qquad (i=1, 2, \dots, n).$$

(For the details, see W. Maak [3].)

Theorem 12. For an almost periodic function on a semi-group, and any element x of G,

$$f(axb) \in U$$

implies

$$f(x) \in U.$$

Proof. Let V be a nbd of E, and $\{A_i\}$ a minimal decomposition of G for U. From Proposition 5.2, we can find A_i and h_i' of G such that

$$x \in A_i, \qquad ah_i'b \in A_i.$$

Hence

$$f(x) = \{f(x) - f(ah_i'b)\} + f(ah_i'b) \in V + U$$

this shows $f(x) \in U$.

From Theorem 12, we have the following

Corollary 12.1. Let $f(x)$ be almost periodic on a semi-group G. For any nbd U, let $\{A_i\}$ be a minimal decomposition of G. Then $a, b \in G$ and $x, y \in A_i$ implies

$$f(axb) - f(ayb) \in U.$$

By Proposition 5.2 and Corollary 12.1, we have

Theorem 13. Let $f(x)$ be almost periodic on a semi-group G. For any nbd U, and x, a, b of G, there is an element x' such that

$$f(cxd) - f(cax'bd) \in U$$

for every c, d of G.