161. On Singular Cross Sections

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1. The theory of obstructions to extentions and homotopies developed by Paul Olum in the paper [3] is generalized to that of cross sections of fibre spaces. Our purpose of the present paper is to give the definition of singular cross sections and their obstruction cocycles, and to state some theorems concerning extentions of cross sections without the proof.

2. Let (X, p, B) be a pseudo fibre space such that the total space X is arcwise connected (p. 63 of [1]; p. 443 of [4]). The projection $p: X\to B$ induces a singular mapping $p: S(X)\to S(B)$ of the singular complex of X into the singular complex of B ((7.1) of [3]). Let S' be a subcomplex of $S(B)$. *A singular cross section φ over S' is defined to be a singular mapping $\varphi: S'\to S(X)$ which satisfies the condition:*

(1) $$\bar{p}\varphi = 1.$$

Let (X, p, K) be a pseudo fibre space such that X is arcwise connected and K is a CW-complex [6]. Let L be a subcomplex of K and let $\varphi: \bar{K}^n = K^n \cup L \to X$ be a cross section. The map φ induces a singular cross section $\bar{\varphi}: S(L)\cup S^n(K)\to S(X)$, where $S^n(K)$ is the n-section of $S(K)$.

Theorem 1. A cross section $\varphi: \bar{K}^n\to X$ is extended over $\bar{K}^{n+1}$ if and only if the singular cross section $\bar{\varphi}: S(L)\cup S^n(K)\to S(X)$ induced by φ is extended over $S(L)\cup S^{n+1}(K)$.

3. Let $\bar{B}=B\times I$ be the Cartesian product of a given space B and the unit interval I. We identify B with the subspace $B\times 0$; then, $S(B)$ is a subcomplex $S_0=S(\bar{B}\times 0)$ of $S(\bar{B})$. Let $S_1=S(B\times 1)$. Define maps $\rho_B: \bar{B}\to B$, $\sigma_B: \bar{B}\to I$ by

$$\rho_B(b, t)=b, \qquad \sigma_B(b, t)=t.$$

The map ρ_B induces a singular mapping $\bar{\rho}_B: S(\bar{B})\to S(B)$.

Let (X, p, B) be as in §2. The spaces $\bar{X}=X\times I$, $\bar{B}=B\times I$ together with a map $q: \bar{X}\to \bar{B}$ defined by $q(x, t)=(p(x), t)$ form a pseudo fibre space $(\bar{X}, q, \bar{B})$. Let $\varphi_0, \varphi_1: S(\bar{B})\to S(X)$ be singular cross sections such that φ_0 agrees with φ_1 over a subcomplex S' of $S(B)$. Let $\bar{S}'$ be a subcomplex of $S(\bar{B})$ defined as in §4 of [3]. *If there is a singular cross section $\Phi: S_0\cup \bar{S}'\cup S^{n+1}(B)\cup S_1\to S(\bar{X})$ such that*