159. *Cohomology of the Three-fold Symmetric Products of Spheres*

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§1. **Introduction.** Let K be a space, and denote by $K^n=K \times K \times \cdots \times K$ the n-fold Cartesian product of K. Then we may regard the symmetric group $\mathfrak{S}_n$ of degree n as a transformation group acting on K^n in a natural fashion as follows: For any $\gamma \in \mathfrak{S}_n$ and $(x_1, x_2, \dots, x_n) \in K^n$, we set $\gamma(x_1, x_2, \dots, x_n)=(x_{\gamma(1)}, x_{\gamma(2)}, \dots, x_{\gamma(n)})$. The orbit space over K^n relative to $\mathfrak{S}_n$ will be called *the n-fold symmetric product* of K.

In the present paper, we shall determine the cohomology of the 3-fold symmetric product $S^n * S^n * S^n$ of an n-sphere $S^n (n \geqq 1)$, by making use of the results and arguments in the previous paper.[1] Full details will appear in the Journal of the Institute of Polytechnics, Osaka City University.

§2. **Methods for calculations.** Let

$$T, S: S^n \times S^n \times S^n \rightarrow S^n \times S^n \times S^n$$

be transformations given by

$$T(x_1, x_2, x_3)=(x_2, x_3, x_1),$$
$$S(x_1, x_2, x_3)=(x_2, x_1, x_3), \qquad (x_1, x_2, x_3 \in S^n)$$

respectively. Then the orbit space over $S^n \times S^n \times S^n$ relative to T is the 3-fold cyclic product ϑ_{n3} of S^n,[2] whose cohomology has determined in CP. Since $TS=ST^2$, $T^2S=ST$, the transformation $S: S^n \times S^n \times S^n \rightarrow S^n \times S^n \times S^n$ induces a transformation $\bar{S}: \vartheta_{n3} \rightarrow \vartheta_{n3}$ such that $\pi S=\bar{S}\pi$, where $\pi: S^n \times S^n \times S^n \rightarrow \vartheta_{n3}$ is the natural projection. Then $\bar{S}$ is the transformation of period 2, and the orbit space over ϑ_{n3} relative to $\bar{S}$ is the symmetric product $S^n * S^n * S^n$. Note that the set of fixed points under $\bar{S}$ is homeomorphic with $S^n \times S^n$. We shall now apply the theory in §1 of CP to the complex ϑ_{n3} with the transformation $\bar{S}$. Then we obtain the results stated in the following.

§3. **The mod 2 cohomology.** The cohomology groups $H^r(S^n * S^n * S^n; Z_2)$ with coefficients in Z_2 are as follows:[3]

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1) Nakaoka, M.,: Cohomology of the p-fold cyclic products, Proc. Japan Acad., **31** (1955). We refer to this paper as CP.

2) This is the notation used in Liao, S. D.,: On the topology of cyclic products of spheres, Trans. Amer. Math. Soc., **77** (1954).

3) We shall write Z and Z_p respectively for the group of integers and the group of integers mod p.