156. A Note on Galois Theory of Division Rings of Infinite Degree

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(Comm. by Z. SUETUNA, M.J.A., Dec. 12, 1955)

Although several generalizations of the theory of Galois for fields have been undertaken for non-commutative fields and rings under some finiteness assumptions,[1] there are few papers concerning non-commutative Galois theory for infinite cases among which one could only mention a work of G. Köthe[2] as a representative one. Recently, N. Nobusawa[3] constructed his Galois theory for division rings of infinite degree: Let $\mathfrak{G}$ be the maximal group of D/L, where D is a division ring and L is a division subring of D. If $\mathfrak{G}$ satisfies the following condition

$(*)$ for each $a \in D$ the set $\{aG;\ G \in \mathfrak{G}\}$ is finite,

then, introducing the same topology as in the theory of Krull for fields,[4] $\mathfrak{G}$ becomes a compact group and there exists the one-to-one correspondence between division subrings of D containing L and closed regular subgroups of $\mathfrak{G}$.

The purpose of the present note is to prove the following

Theorem. *Let D be a division ring, L be a division subring and $\mathfrak{G}$ be the maximal group of D/L. If $\mathfrak{G}$ satisfies the following condition*

$(*)$ *for each $a \in D$ the set $\{aG;\ G \in \mathfrak{G}\}$ is finite,*

then there hold the next propositions:

(i) *If C, the center of D, is infinite then $V_D(L)=C$.*[5]

(ii) *If $V_D(L) \supsetneqq C$ then $V_D(L)$ is finite.*

To prove this theorem, we shall require a chain of lemmas, which will be stated in the form rather general.

Throughout the paper, R will be a simple ring (i.e. a primitive ring with minimum condition), R' be a simple subring of R (with

1) G. Azumaya: Galois theory for uniserial rings, J. Math. Soc. Japan, **1** (1949). H. Cartan: Théorie de Galois pour les corps non commutatifs, Ann. Ecole Norm. Sup., **64** (1947). J. Dieudonné: La théorie de Galois des anneaux simples et semi-simples, Comm. Math. Helv., **21** (1948). N. Jacobson: A notè on division ring, Amer. J. Math., **69** (1947). T. Nakayama: Galois theory of simple rings, Trans. Amer. Soc., **73** (1952).

2) G. Köthe: Schiefkörper unendlichen Ranges über dem Zentrum, Math. Ann., **105** (1931).

3) N. Nobusawa: An extension of Krull's Galois theory to division rings, Osaka Math. J., **7** (1955).

4) W. Krull: Galoissche Theorie der unendlichen algebraischen Erweiterungen, Math. Ann., **100** (1928).

5) $V_D(L)$ denotes the centralizer of L in D.