6. On the Convergence Character of Fourier Series. II

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1. Let $f(x)$ be an integrable function with period 2π and $s_n(x)$ be the nth partial sum of its Fourier series. S. Izumi[1] has proved the following

Theorem I. *If $f(x)$ belongs to the Lip $\alpha(0<\alpha\leqq 1)$ class, then the series*

$$\sum_{n=2}^{\infty} | s_n(x)-f(x) |^2/n^\beta (\log n)^\gamma$$

converges uniformly, where $\beta=1-2\alpha$ and $\gamma>1$ or >2, according as $0<\alpha<1/2$ or $1/2\leqq\alpha\leqq 1$.

In a previous paper,[2] we have shown that Theorem I is still valid even if the restriction $\gamma>2$ is replaced by $\gamma>1$ for $\alpha=1/2$. The object of this paper is to show that the restriction $\gamma>2$ in Theorem I may be replaced by $\gamma>1$ for $\alpha\geqq 1/2$. In fact we prove

Theorem 1. *Let $1\geqq\alpha>0$ and $k>0$. If $f(x)$ belongs to the Lip α class, then the series*

$$\sum_{n=2}^{\infty} \frac{| s_n(x)-f(x) |^k}{n^\delta (\log n)^\gamma}$$

converges uniformly, where $\delta=1-k\alpha$ and $\gamma>1$.

Proof of Theorem 1.[3] we have

$$s_n(x)-f(x)=\frac{1}{\pi}\int_0^\pi \varphi_x(t)\sin (n+1/2)t/\{2\sin t/2\}\, dt$$

$$=\frac{1}{\pi}\int_0^\pi \varphi_x(t)p(t)\sin nt\, dt+\frac{1}{2\pi}\int_0^\pi \varphi_x(t)\cos nt\, dt,$$

$$=P_n(x)+Q_n(x),$$

where $\varphi_x(t)=\varphi(t)=f(x+t)+f(x-t)-2f(x)$ and $p(t)=\cos t/2/\{2\sin t/2\}$.

We may take a number p' such that $p'\geqq 2$, $p'\geqq k$ and $p'>1/\alpha$ for given α and k.

By the Hausdorff-Young inequality, we get[4]

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1) S. Izumi: Some trigonometrical series. III, Proc. Japan Acad., **31**, 257–260 (1955).

2) M. Kinukawa: On the convergence character of Fourier series, Proc. Japan Acad., **31**, 513–516 (1955).

3) M. Kinukawa: Some strong summability of Fourier series (to appear).

4) A denotes an absolute constant, which may be different in each occurrence, and p' denotes the conjugate number of p, that is, $1/p+1/p'=1$.