1. *Evans's Theorem on Abstract Riemann Surfaces with Null-Boundaries.* I

By Zenjiro KURAMOCHI

Mathematical Institute, Osaka University

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G. C. Evans[1)] proved the following

Evans's theorem. *Let F be a closed set of capacity zero in the 3-dimensional euclidean space (or z-plane). Then there exists a positive unit-mass-distribution on F such that the potential engendered by this distribution has limit ∞ at every point of F.*

Let R^* be a null-boundary Riemann surface and let $\{R_n\}$ $(n=0, 1, 2, \cdots)$ be its exhaustion with compact relative boundaries $\{\partial R_n\}$. Put $R=R^*-R_0$. After R. S. Martin,[2)] we introduce ideal boundary points as follows. Let $\{p_i\}$ be a sequence of points of R tending to the ideal boundary of R and let $\{G(z, p_i)\}$ be Green's function of R with pole at p_i. Let $\{G(z, p_{i_j})\}$ be a subsequence of $\{G(z, p_i)\}$ which converges to a function $G(z, p)$ uniformly in R. We say that $\{p_{i_j}\}$ determines a Martin's point p and we make $G(z, p)$ correspond to p. Furthermore Martin defined the distance between two points p_1 and p_2 of R or of the boundary by

$$\delta(p_1, p_2)=\sup_{z\in R_1-R_0}\left|\frac{G(z, p_1)}{1+G(z, p_1)}-\frac{G(z, p_2)}{1+G(z, p_2)}\right|.$$

It is clear that Martin's point p coincides with an ordinary point when $p\in R$ and that if $p_i\overset{\mathfrak{M}}{\rightarrow} p$,[3)] $G(z, p_i)\rightarrow G(z, p)$ uniformly in R. In the following, we denote by $\overline{R}$[4)] the sum of R and the set B of all ideal boundary points of Martin. Let p be a point of $\overline{R}$ and let $V_m(p)$ be the domain of R such that $\varepsilon[G(z, p)\geqq m]$. Then

Lemma 1. $\displaystyle\int_{\partial V_m(p)}\frac{\partial G(z, p)}{\partial n}ds=2\pi$:[5)] $\quad m\geqq 0.$

Proof. Let $p=\overset{\mathfrak{M}}{\lim\limits_i}\, p_i$: $p\in B$, $p_i\in R$. Then $\underset{R-V_m(p_i)}{D}[G(z, p_i)]=2\pi m$ and

1) G. C. Evans: Potential and positively infinite singularities of harmonic functions, Monatschefte Math. U. Phys., **43** (1936).

2) R. S. Martin: Minimal positive harmonic functions, Trans. Amer. Math. Soc., **49** (1941).

3) In this paper $\mathfrak{M}$ means "with respect to Martin's metric".

4) The topology induced by this metric restricted in R is homeomorphic to the original topology and it is clear that B and $\overline{R}$ are closed and compact.

5) In this article, we denote by ∂A the relative boundary of A.