28. A Remark on the Ranged Space

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1. In this paper, we shall change a given system of neighbourhoods which, in general, does not satisfy the axiom (C) of Hausdorff:

(C) let $v(p)$ be an arbitrary neighbourhood of p and let q be an arbitrary point of $v(p)$, there exists a neighbourhood $v(q)$ such as $v(q)\subseteq v(p)$,

in order to make a new system satisfy the axiom (C), and to make the new system be equivalent to the old one provided the latter satisfies the axiom (C). Then we apply this fact to the ranged space[1] and show that the axiom (C) is needless to define the ranged space.

Definition 1. Consider a space R where the topology T is given by a system $\{v\}$ of neighbourhood satisfying the axiom (A); $p\in v(p)$, and $\{v(p)\}\neq\phi$ for any point p of R. When a set $A\subseteq R$ is given, we shall denote by $\tilde{A}$ the set of all points p which have the following property:

When an arbitrary neighbourhood $v(p)$ of p is given, there exist a point $a\in A$ and a neighbourhood $v(a)$ such as $v(a)\subseteq v(p)$. And we shall denote a new topology, which has $\tilde{A}$ as the closure of A, by T^*. Then it is clear that $\bar{A}\supseteq\tilde{A}$, where $\bar{A}$ means the closure of A with respect to the old topology T.

Theorem 1. *The topology T^* has the following properties*:

(I) $$A\subseteq\tilde{A},$$

(II) $$\text{if } A\subseteq B, \text{ then } \tilde{A}\subseteq\tilde{B},$$

(III) $$\tilde{\phi}=\phi,$$

(IV) $$\tilde{\tilde{A}}\subseteq\tilde{A}.$$

Proof. (I) is evident. Let $A\subseteq B$ and let p and $v(p)$ be an arbitrary point $\tilde{A}$ and an arbitrary neighbourhood of p respectively, there exist a point $a\in A$ and a neighbourhood $v(a)$ such as $v(a)\subseteq v(p)$. From $A\subseteq B$ it follows that $a\in B$, therefore $p\in\tilde{B}$. (III) follows from the fact that $\bar{\phi}=\phi$ and T^* is stronger than T. Let p and $v(p)$ be

1) K. Kunugi: Sur les espaces complets et régulièrement complets. I–II, Proc. Japan Acad., **30** (1954).