27. On the Property of Lebesgue in Uniform Spaces. VI

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(Comm. by K. KUNUGI, M.J.A., Feb. 13, 1956)

Let S be a topological space. A covering of S is a family of open sets whose union is S. A covering is called *finite*, if it consists of a finite family.

Let us consider a *separated* uniform space S with a filter of surroundings $\mathfrak{S}$. A covering $\mathfrak{F}$ of S is said to have the *Lebesgue property* if there is a surrounding V in $\mathfrak{S}$ such that, for each point x of S, we can find an open set 0 of $\mathfrak{F}$ satisfying $V(x)\subset 0$.

We say that a separated uniform space has the *finite Lebesgue property* if any finite covering has the Lebesgue property. If any covering of S has the Lebesgue property, the space S is said to have the Lebesgue property. Such a space was studied by K. Iséki [4] and S. Kasahara [5]. S. Kasahara ([5], p. 129) has proved that *every uniform space having the Lebesgue property is complete.* On the other hand, the present author ([4], V, p. 619) has shown that the finite Lebesgue property does not imply the Lebesgue property and the existence of a non-complete uniform space having the finite Lebesgue property.

In this Note, we shall prove the following

Theorem 1. If the completion of a uniform space having finite Lebesgue property is normal, it has the finite Lebesgue property.

As easily seen, the converse of Theorem 1 is not true. There are non-normal complete uniform spaces (J. Dieudonné [2]).

To prove this suppose that $\hat{S}$ is the completion of a uniform space S having the finite Lebesgue property. According to a theorem of my Note ([4], IV, p. 524), it is sufficient to prove the following proposition.

Every bounded continuous function on $\hat{S}$ is uniformly continuous.

Let $f(x)$ be a continuous function on $\hat{S}$, then the restricted function $f(x|S)$ on S is uniformly continuous. Therefore, $f(x|S)$ is uniform continuously extended on $\hat{S}$ and it coincides with $f(x)$. Thus $f(x)$ is uniformly continuous, and $\hat{S}$ has the finite Lebesgue property.

Under the assumption of Theorem 1, we shall consider the relation between the dimension of S and its completion $\hat{S}$. There are some definitions of dimension for a topological space. However,