21. Some Trigonometrical Series. XIX

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1. In the preceding paper [1], we have proved the following

Theorem 1.[1] *If* $p \geqq \lambda > 1$, $\varepsilon > 0$ *and*

$$\left(\int_0^{2\pi} |f(x+t)-f(x-t)|^p dx\right)^{1/p} = O\left(t^{1/\lambda} \Big/ \left(\log \frac{1}{t}\right)^{(1+\varepsilon)/\lambda}\right),$$

then the series

$$\sum |s_n(x)-f(x)|^\lambda$$

converges almost everywhere, where $s_n(x)$ *denotes the nth partial sum of the Fourier series of* $f(x)$.

We shall here consider the case $\lambda=1$ and in fact prove the following

Theorem 2.[2] *If* $f(x)$ *is differentiable almost everywhere and*

$$(1) \qquad \left(\int_0^{2\pi} |f'(x+t)-f'(x-t)|^p dx\right)^{1/p} \leqq A \Big/ \left(\log \frac{1}{t}\right)^\beta$$

where $p>1$ *and* $\beta>1$, *then the series*

$$(2) \qquad \sum |s_n(x)-f(x)|$$

converges almost everywhere.

More generally, the condition (1) may be replaced by

$$\sum_{n=1}^{\infty} n^{-1} \omega'_p(n^{-1}) < \infty$$

where

$$\omega'_p(t) = \max_{0 \leqq h \leqq t} \left(\int_0^{2\pi} |f'(x+h)-f'(x-h)|^p dx\right)^{1/p}.$$

The method of proof is similar to that of [1].

2. For the proof of Theorem 2 we need a lemma due to A. Zygmund [2]:

Lemma. *Suppose that* $p>1$ *and*

$$\left\| \sum_{\nu=m}^{n} \gamma_\nu e^{i\nu x} \right\|_p \leqq C$$

where $\| \ \|_p$ *denotes the* L^p*-norm and suppose that*

$$|\lambda_\nu| \leqq M, \quad \sum_{\nu=m}^{n-1} |\lambda_\nu - \lambda_{\nu+1}| \leqq M,$$

1) In [1], it is written that $p \geqq \lambda \geqq 1$, but the case $\lambda=1$ is trivial. The assumption that "$f(t)$ is of the power series type", and its foot-note are superfluous.

2) G. Sunouchi and T. Tsuchikura remarked the author that the case $p=2$ is equivalent to a theorem of Tsuchikura [4].