18. *Note on Leontief's Dynamic Input-Output System*[1)]

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Leontief [2] raises the problem whether there exists a solution of the dynamic input-output system, when a flexible accelerator is introduced so that demand for capital equipment is proportional to the rate of change of the output when the latter is rising but zero when it is falling. In this paper, we shall concern ourselves with the condition, which must be imposed upon the input matrix and the stock matrix in order to exist a solution of the system.

1. Let there be a closed economy with n industries, $x_i(t)$ be the annual rate of total output of industry i, and $s_{ik}(t)$ be the stock of a commodity produced by industry i and used by industry k at time t. Then, if we suppose that all kinds of stocks are *irreversible*, *Leontief's dynamic input-output system* can be represented by the following (*)-*system* of differential equations:[2)]

$$(*)\qquad \begin{cases} x_i - \sum_{k=1}^{n} a_{ik}x_k = \sum_{k=1}^{n} \dot{s}_{ik}, \quad (i=1,\cdots,n), \\ \dot{s}_{ik} = \begin{cases} b_{ik}\dot{x}_k, & s_{ik}=b_{ik}x_k \text{ and } \dot{x}_k>0, \\ 0, & s_{ik}=b_{ik}x_k \text{ and } \dot{x}_k\leqq 0, \\ 0, & s_{ik}>b_{ik}x_k, \quad (i,k=1,\cdots,n), \end{cases} \end{cases}$$

where a_{ik} is the *input coefficient*, i.e. the amount of the product of industry i absorbed annually by industry k per unit of x_k, and b_{ik} is the *capital coefficient*, i.e. the stock of the product of industry i used per unit of the annual output of industry k. Upon these coefficients, we impose the following restrictions:

$$a_{ik}\geqq 0,\ b_{ik}\geqq 0,\quad (i,k=1,\cdots,n),$$

$$\sum_{k=1}^{n} a_{ik}\leqq 1,\quad (i=1,\cdots,n),$$

$$\sum_{k=1}^{n} a_{ik}<1 \text{ at least for one } i.$$

By using the matricial notation, we may write the first equation as follows:

$$(I-A)x=\dot{S}\begin{bmatrix}1\\ \vdots \\ 1\end{bmatrix}$$

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2) $\dot{s}_{ik}=\frac{ds_{ik}}{dt}$ will stand for the rate of change of the variable s_{ik} with respect to time t.