37. *On Closed Mappings and Dimension*

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1. Introduction. Let X be a normal space. We shall denote by "$\dim X$" the covering dimension of X and by "$\operatorname{ind}\dim X$" the inductive dimension of X which is defined by separation of closed sets; $\dim X \leqq n$ if every finite open covering of X has an open refinement of order $\leqq n+1$, and $\operatorname{ind}\dim X \leqq n$ if for any pair of a closed set F and an open set G with $F \subset G$ there exists an open set V such that $F \subset V \subset G$, $\operatorname{ind}\dim(\overline{V}-V) \leqq n-1$, where by definition $\operatorname{ind}\dim X = -1$ if and only if X is empty.

In this paper we shall establish the following generalizations of W. Hurewicz's theorems.[1]

Theorem 1. *Let f be a closed continuous mapping of a normal space X onto a normal space Y such that the inverse image $f^{-1}(y)$ consists of at most $k+1$ points for each point y of Y. Then we have*

$$\dim Y \leqq \operatorname{ind}\dim X + k.$$

Theorem 2. *Let f be a closed continuous mapping of a normal space X onto a paracompact T_1-space Y such that*

$$\dim f^{-1}(y) \leqq m$$

for each point y of Y. Then

$$\dim X \leqq \operatorname{ind}\dim Y + m.$$

2. Lemmas. Let $\mathfrak{G}$ be an open covering of a space X and A a subset of X. We shall write $(\mathfrak{G})$-$\dim A \leqq n$ if there exists an open covering of a subspace A which has an order $\leqq n+1$ and is a refinement of $\mathfrak{G}$.

Lemma 1. *Let X be a normal space. Then we have* $\dim X \leqq n$ *if and only if, for any pair of a closed set F and an open set G with $F \subset G$ and for any finite open covering $\mathfrak{G}$ of X, there exists an open set V such that*

$$F \subset V \subset G, \quad (\mathfrak{G})\text{-}\dim(\overline{V}-V) \leqq n-1.$$

This is proved in [4]. From this lemma we get immediately Lemma 2 which is due to N. Vedenisoff.

Lemma 2. *If X is a normal space, then we have*

$$\dim X \leqq \operatorname{ind}\dim X.$$

In case A is a closed subset of a normal space X, we shall

1) W. Hurewicz proved these theorems for the case where X and Y are separable metric spaces. Cf. [2], [3]. In [7] we have used Theorem 1 for the case of metric spaces.