58. Note on the Lebesgue Property in Uniform Spaces. II

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Let E be a separated uniform space. An open covering $\mathfrak{S}$ of E is said to have the *Lebesgue property* if there exists a surrounding V of E such that, for each point $x \in E$, we can find a member O of the covering $\mathfrak{S}$ containing the set $V(x)$. If every covering of E consisting of a countable number of open sets (such a covering is called a *countable open covering*) has the Lebesgue property, the space E is said to have the *countable Lebesgue property.* All uniform spaces to be considered in what follows will be assumed separated.

In a previous note [4], it has been shown among other things that a uniformisable Hausdorff space possesses a compatible uniform structure for which every open covering of the space has the Lebesgue property if and only if it is paracompact, and the corresponding proposition to the finite Lebesgue property.[1]

A purpose of this note is to prove an analogous proposition for the countable Lebesgue property, and to give a characterization of a uniform space having the countable Lebesgue property.

First of all we can easily obtain the following

THEOREM 1. *Let E be a uniformisable Hausdorff topological space, then E possesses a uniform structure compatible with the topology of E for which E has the countable Lebesgue property if and only if E is countably paracompact*[2] *and normal.*

It is known[3] that a uniform space having the countable Lebesgue property is countably paracompact and normal, and so we have only to prove the if part of the theorem. Now, by a theorem of C. H. Dowker [1] and a theorem of K. Morita [5], we can conclude that every countable open covering of a countably paracompact normal space is normal[4] and the normal sequence consists of countable open coverings.

Therefore, the family of all countable open coverings of E forms

1) A separated uniform space is said to have the *finite Lebesgue property* if every covering consisting of a finite number of open sets has the Lebesgue property.

2) A Hausdorff topological space E is *countably paracompact* if every countable open covering of E has a locally finite refinement. Cf. C. H. Dowker [1].

3) Cf. S. Kasahara [3].

4) A sequence $\{\mathfrak{S}_n\}$ of coverings of a space is called a *normal sequence* if, for each n, the covering $\mathfrak{S}_{n+1}$ is a star refinement of $\mathfrak{S}_n$. A covering $\mathfrak{S}$ is normal if a normal sequence $\{\mathfrak{S}_n\}$ exists such that $\mathfrak{S}_1$ is a refinement of $\mathfrak{S}$. Cf. J. W. Tukey [7].