57. On Some Types of Polyhedra

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Let Q be a class of spaces having some topological property. According to O. Hanner [1], a space Y is called respectively an extensor for Q-spaces (an $ES(Q)$) or a neighborhood-extensor for Q-spaces (an $NES(Q)$) if every Y-valued mapping (=continuous transformation) defined on any closed subset C of any Q-space X always allows a continuous extension to the whole space X or to an open set G which contains C; and a space Y is called respectively an absolute retract for Q-spaces (an $AR(Q)$) or an absolute neighborhood retract for Q-spaces (an $ANR(Q)$) if Y is a Q-space and is a retract or a neighborhood retract of any Q-space containing Y as a closed subset. Analogously to these definitions we generalize the definition of an absolute n-retract which was given by C. Kuratowski [2] as follows: A space Y is called an n-$ES(Q)$ if every Y-valued mapping defined on any closed subset C of any Q-space X with an arbitrary small open set $G\supset C$ such that $\dim(X-G)\leqq n$, always allows a continuous extension to the whole space X; and a space Y is called an n-$AR(Q)$ if Y is a Q-space and is a retract of any Q-space or X containing Y as a closed subset where $\dim(X-G)\leqq n$ holds for an arbitrary small open set $G\supset C$. When Q is a class of metric spaces or of normal spaces, a Q-space Y which is an n-$ES(Q)$ is an n-$AR(Q)$ and conversely: This is essentially proved in [1, Theorem 8.1]. An n-sphere is a well-known example which is an n-AR (normal) [3, Theorem 6.1]. We shall study some types of polyhedra which are n-$ES(Q)$ for the case when Q is a class of metric spaces or of normal spaces.

Let $P=\{p_\alpha\}$ be an abstract set of points with $|P|\geqq n+1$, which will be called a vertex-set, where n is an arbitrary positive integer. The complex with the weak topology spanned by all m-simplexes, $m\leqq n$, whose vertices are mutually different points of P is called an *n-full-polyhedron* based on P and is denoted by $K(n, P)$. An n-sphere or an n-simplex is respectively nothing but an n-full-polyhedron based on P with $|P|=n+2$ or with $|P|=n+1$.

Theorem 1. *An n-full-polyhedron $K(n, P)$ is an n-ES (metric) for an arbitrary infinite vertex-set $P=\{p_\alpha; \alpha\in A\}$.*

Proof. Let X be a metric space, C be a closed subset of X with an arbitrary small open set D with $\dim(X-D)\leqq n$ and f be