56. On Semi-reducible Measures. II

By Tadashi ISHII
Department of Mathematics, Ehime University, Japan

(Comm. by Z. SUETUNA, M.J.A., April 12, 1956)

In this note we show that main results concerning semi-reducibility of Baire (Borel) measures, which have been proved by Marczewski and Sikorski [5] in metric spaces, and by Katětov [4, Theorem 1] and the present author [3, Theorem 4] in paracompact spaces, are valid in completely regular spaces with a complete structure.[1] The case of two-valued measures has already been considered by Shirota [6], though his result is related to Q-spaces of Hewitt [1]. We use the same notations as in the previous paper [3]: $\mathfrak{B}^*(X)$=all of Baire subsets in a T-space $X, C(X,R)$=all of real-valued continuous functions on $X, P(f)=\{x\mid f(x)>0, f\in C(X,R)\}$, $\mathfrak{P}(X)=\{P(f)\mid f\in C(X,R)\}$.

All spaces considered are completely regular spaces and all measures considered are finite measures, unless the contrary is explicitly stated.

Lemma 1. *If any closed discrete subset in a T_1-space X has the power of (two-valued) measure 0,[2] then for any (two-valued) Baire measure μ in X, the union of a discrete collection of open subsets $\{G_\alpha \mid G_\alpha\in\mathfrak{P}(X), \mu(G_\alpha)=0\}$ has also μ-measure 0.*[3]

Since the proof is essentially stated in the previous paper [3, Theorem 4], we do not repeat it here.

Lemma 2. *Let $\mathfrak{U}=\{U_\alpha\mid\alpha\in A\}$ be a normal covering of a T-space X. Then there exists a refinement $\mathfrak{B}=\{G_{n\alpha}\mid\alpha\in A,\ n=1,2,\cdots\}$ of $\mathfrak{U}$ such that $\{G_{n\alpha}\mid\alpha\in A\}$ is a discrete collection with $G_{n\alpha}\in\mathfrak{P}(X)$ for each n.*

Proof. Let $\mathfrak{U}=\{U_\alpha\mid\alpha\in A\}$ be a normal covering of X and let $\{\mathfrak{U}_n\}$ be a normal sequence such that $\mathfrak{U}_1\overset{\Delta}{>}\mathfrak{U}_2\overset{\Delta}{>}\cdots\overset{\Delta}{>}\mathfrak{U}_n\overset{\Delta}{>}\cdots$. Then, as Stone [7] has showed, there exists a closed covering $\{F_{n\alpha}\mid\alpha\in A, n=1,2,\cdots\}$ satisfying the following conditions:

i) $S(F_{n\alpha},\mathfrak{U}_{n+3})\cap S(F_{n\gamma},\mathfrak{U}_{n+3})=\phi$ if $\alpha\neq\gamma$,

ii) $\{F_{n\alpha}\mid\alpha\in A\}$ is a discrete collection for each n,

1) A measure μ defined on a σ-field $\mathfrak{B}$ containing Baire family in a T-space is called semi-reducible if there exists a closed subset Q such that (1) $\mu(G)>0$ holds if G is open, $G\in\mathfrak{B}$, $G\frown Q\neq\phi$, and (2) $\mu(F)=0$ holds if F is closed, $F\in\mathfrak{B}$, $F\frown Q=\phi$.

2) A discrete set is called to have the power of (two-valued) measure 0, if every (two-valued) measure, defined for all subsets and vanishing for all one point, vanishes identically.

3) A collection $\{H_\alpha\mid\alpha\in A\}$ of subsets of a T-space is called discrete if (1) the closures $\overline{H}_\alpha$ are mutually disjoint, (2) $\cup_{\beta\in B}\overline{H}_\beta$ is closed for anv subset B of A.