77. On $H_*(\Omega^N(S^n); Z_2)$

By Tatsuji KUDO and Shôrô ARAKI
Faculty of Science, Kyusyu University
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§1. In this note we shall give a brief account about the determination of the modulo 2 Pontrjagin ring $H_*(\Omega^N(S^n); Z_2)$ of the N-times iterated loop space $\Omega^N(S^n)$ of the n-sphere S^n, where $0<N<n$.

For this purpose we first introduce a new concept of an H_n-space, of which the $(n+1)$-times iterated loop space of a metrizable space is a typical example. Then we define some homological operation modulo 2, which may, at least formally, be regarded as dual to the Steenrod's squaring operations. In fact, although they are defined only within the category of H_n-spaces, necessary transgression theorems in the homology theory may be established with respect to these operations.

The complete discussions of our note will be published in a forthcoming Memoirs of the Faculty of Science, Kyusyu University.

§2. **H_n-spaces. Definition 1.** We say that a space X has an *H_n-structure* (or is an *H_n-space*), when there exists a system of maps $\{\theta_m\}$, $(0\leqq m\leqq n)$ subject to the following conditions:

(i) θ_m is a map

(1.a)$_m$ $$\theta_m: I^m\times X\times X\to X$$

where I is a unit interval and I^m the m-fold product of I; in particular

(1.a)$_0$ $$\theta_0: X\times X\to X.$$

(ii) θ_m's satisfy

(1.b) $$\theta_m(t_1,\cdots,t_{i-1},0,t_{i+1},\cdots,t_m;x,y)=\theta_{i-1}(t_1,\cdots,t_{i-1};x,y),$$ $$\theta_m(t_1,\cdots,t_{i-1},1,t_{i+1},\cdots,t_m;x,y)=\theta_{i-1}(1-t_1,\cdots,1-t_{i-1};y,x),$$

for any m, i, $(t_1,\cdots,t_m)\in I^m$, and $x,y\in X$.

(iii) There exists an element $e\in X$, called the *unit* of this H_n-structure, satisfying

(1.c) $$\theta_m(t_1,\cdots,t_m;x,e)=\theta_m(t_1,\cdots,t_m;e,x)=x$$

for any $(t_1,\cdots,t_m)\in I^m$ and $x\in X$.

For an H_n-space X, θ_0 defines a product on X, and we may consider X as an H-space in the widest sense. It is called *homotopy-associative* if it is so when regarded as an H-space.

Let X be an H_n-space and $X_1=\Omega(X)$ the space of loops in X with e as the reference point, and X_1' be the space of paths ending at e, with the usual topology.

Proposition 1. *If X is an H_n-space, then X_1 and X_1' are also H_n-spaces.*