75. Notes on Topological Spaces. III. On Space of Maximal Ideals of Semiring

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S. Bourne [1] considered the Jacobson radical of a semiring and recently W. Slowikowski and W. Zawadowski [6] developed the general theory and space of maximal ideals of a positive semiring. R. S. Pierce [4] considered a topological space obtained from a semiring. A. A. Monteiro [3] wrote an excellent report on representation theory of lattices.

Definition 1. A *semiring* A is an algebra with two binary operations, addition (written $+$) which is associative, and multiplication which is associative, and satisfies the distributive law

$$a(b+c)=ab+ac, \quad (b+c)a=ba+ca.$$

In this paper, we suppose that A has the further properties:

1) There are two elements $0, 1$ such that

$$x+0=x, \qquad x\cdot 1=x$$

for every x of A.

2) Two operations, addition and multiplication, are commutative.

Definition 2. A non-empty proper subset I of A is called an *ideal*, if

(1) $a, b \in I$ implies $a+b \in I$,

(2) $a \in I, x \in A$ implies $ax \in I$.

W. Slowikowski and W. Zawadowski [6] proved that *every ideal is contained in a maximal ideal.* An ideal is maximal if there is no ideal containing properly it.

Let $\mathfrak{M}$ be the set of all maximal ideals in a semiring A. We shall define two topologies on $\mathfrak{M}$.

For every x of A, we denote by Δ_x the set of all maximal ideals containing x, and by Γ_x the set $\mathfrak{M}-\Delta_x$, i.e. the set of all maximal ideals not containing x. Let I be an ideal of A, we denote by Δ_I the set of all maximal ideals containing I.

We shall choose the family $\{\Delta_x \mid x \in A\}$ as a subbase for open sets of $\mathfrak{M}$. We shall refer to the resulting topology on $\mathfrak{M}$ as Δ-topology (in symbol, $\mathfrak{M}_\Delta$). Similarly, we shall take the family $\{\Gamma_x \mid x \in A\}$ as a subbase for open sets of $\mathfrak{M}$ (in symbol, $\mathfrak{M}_\Gamma$). These two topologies for normed ring or general commutative ring were considered by I. Gelfand and G. Silov [2] or P. Samuel [5].

Let M_1, M_2 be two distinct elements of $\mathfrak{M}_\Delta$. Then we have $M_1+M_2=A$. Therefore there are a, b such that $a+b=1$ and $a \in M_1$,