73. *Local Properties of Topological Spaces*

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1. Introduction. We say that a topological space R has a certain property P of topological spaces locally if every point of R has a neighborhood with a property P and that P is a *local property* of R. If R is covered by a normal open covering in the sense of J. W. Tukey [5] each element of which has a property P, we say that R has a property P uniformly locally and that P is a *uniform local property* of R. If R itself has a property P, we say that P is a *global property* of R. Any global property P of R is of course a (uniform) local property and conversely there are several important properties P such that if R has P (uniformly) locally R has it globally. It is natural to raise the question:

Under what conditions is a (uniform) local property P of R a global property?

E. Michael [1] has shown that the axiomatic treatment can be applied to this problem, introducing the concepts such as G- and F-hereditary properties. The purpose of this paper is to show that the concepts of CG- or CF-hereditary property P (Definitions 1 and 2 below) can also be applied to this problem and to apply this results to several special cases.

2. General cases. A collection $\{S_\alpha\}$ of subsets of a topological space R is called *discrete* if $\{\overline{S}_\alpha\}$ is locally finite and mutually disjoint.

Definition 1. Let P be a property of topological spaces and R be a topological space. We say that P is a *CG-hereditary property* of R if the following conditions are satisfied:

(G1) If S is a subset of R with a property P, every elementary open[1] subset of S has also a property P.

(G2) If $\{S_\alpha\}$ is a discrete collection of subsets of R each element of which has a property P, $\smile S_\alpha$ has also a property P.

(G3) If $\{S_i\}$ is a countable locally finite collection of elementary open sets of R each element of which has a property P, $\smile S_i$ has also a property P.

A covering $\mathfrak{U}=\{U_\alpha; \alpha \in \smile A_i\}$ of a topological space R is called a strong screen if every $\mathfrak{U}_i=\{U_\alpha; \alpha \in A_i\}$ is discrete and $\{\underset{\alpha\in A_i}{\smile} U_\alpha; i=1, 2, \cdots\}$ is locally finite.

1) A subset S of R is called elementary open if S is of the form $\{x; f(x)>0\}$, where f is continuous.