72. *On a Theorem of Ugaheri*

By Masanori KISHI
Mathematical Institute, Nagoya University

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1. In his paper [6], Ugaheri investigated the potentials in m-dimensional euclidean space R^m, with respect to a kernel function $\Phi(r)$ which is positive, continuous and monotonously decreasing at $r>0$, and he proved that these potentials satisfy the following maximum principle: if the potential of a positive measure with compact carrier is not greater than M on the carrier of the measure, then it is $\leqq kM$ everywhere in R^m, where k is an absolute constant depending only upon the space R^m. Furthermore, using this maximum principle, he proved Evans-Vasilesco's theorem for the potentials with respect to $\Phi(r)$.

In the present note we shall consider the potentials in a locally compact space Ω and prove that the generalized form of the maximum principle of Ugaheri is equivalent to the continuity principle with certain additional condition, where the continuity principle means that, if the potential of a positive measure, considered as a function on the carrier of the measure, is continuous, then it is also continuous in the whole space.

2. We assume that there is given a real-valued function $\Phi(p, q)$, defined in the product space $\Omega\times\Omega$ and satisfying the following conditions:

1° $\Phi(p, q)$ is positive, continuous in (p, q) except for the diagonal set of $\Omega\times\Omega$ and symmetric, that is, $\Phi(p, q)=\Phi(q, p)$:

2° At every point p of Ω, $\lim\limits_{q\to p}\Phi(p, q)=+\infty$ and, in case Ω is not compact, $\lim\limits_{q\to\omega}\Phi(p, q)=0$, where ω is the Alexandroff point.

The potential $U^\mu(p)$ of a positive measure μ is defined by the Radon-Stieltjes integral

$$U^\mu(p)=\int_\Omega \Phi(p, q)\, d\mu(q).$$

We assume the following condition:

3° $\Phi(p, q)$ satisfies the energy principle in the sense of Ninomiya [4].

Let K be a compact subset of Ω, and denote by $\mathfrak{M}_K^1$ the family of all positive measures on K which are of total measure 1. We put

$$W(K)=\inf_{\mu\in\mathfrak{M}_K^1}\int U^\mu\, d\mu,$$