95. A Remark on the Ideal Boundary of a Riemann Surface

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(Comm. by K. KUNUGI, M.J.A., June 12, 1956)

Let W be an open Riemann surface, and BD the class of piecewise smooth functions f defined on W and bounded on it having a finite Dirichlet integral $D[f]$. We say that a sequence $\{f_n\}$, $f_n \in BD$, converges to f in BD if $\{f_n\}$ is uniformly bounded and $f_n \to f$ uniformly on every compact subset of W while $D[f_n - f] \to 0$.

The class of BD functions with compact carriers forms an ideal in the ring BD. If we denote by $\overline{K}$ these functions which are limits in BD of sequences from K, we have the decomposition of $f \in BD$ as follows:

$$f = \varphi + u, \qquad \varphi \in \overline{K}, \quad u \in HBD,$$

where HBD is the class of harmonic functions in BD.

To make use of the theory of normed ring, we introduce in BD a new norm given by

$$\|f\| = \sup_W |f| + \sqrt{D[f]}.$$

Since HBD is complete in this norm, the completion of $\overline{K}$ completes BD. The notation $\overline{K}$ is again used for the completed $\overline{K}$. This completed BD is a normed ring A and the set of the maximal ideals constructs a compact Hausdorff space W^* containing W as a dense subset. $f \in A$ can be represented as a continuous function on W^* and it is equivalent to $f \in M$ that f equals to zero on a point M of W^*.

The maximal ideals, which contain K, form a closed non-dense subset Γ of W that does not correspond to the inner points of W. And we regard it as the ideal boundary of W (Royden [4]). Following the statement of Royden [4] the set Δ of the maximal ideals containing $\overline{K}$ is named the harmonic boundary of W. Δ is a closed subset of Γ, which disappears in the parabolic case.

The existence of $\Gamma - \Delta$ for the hyperbolic case is known in a special case. To see this, we first observe the behaviour of Green's function $g(p, q)$ of W on Δ. By Royden [3]

$$\bar{g}(p, q) = \min[l, g(p, q)] \in \overline{K}.$$

This shows that $\bar{g}(p, q)$ represented as a function of W^* is $\bar{g}(M, q) = 0$ for $M \in \Delta$.

Now we note that a separation of W into disjoint parts by a finite number of compact curves also separates Γ. We assume that W has an end W' bounded by a finite number of compact curves and