94. *Some Classes of Riemann Surfaces Characterized by the Extremal Length*

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In this article we shall consider some classes of Riemann surfaces characterized by the extremal length and state their properties, the detailed proofs of which will be given in another paper[1] together with other related results.

1. Let $\{c\}$ $(\neq\phi)$ be a system of curves each of which consists of a finite or countable number of curves on an arbitrary Riemann surface R. For any non-negative covariant ρ on R such that

$$\underline{\int_c} \rho(z)\,|dz| \geqq 1, \text{ for all } c \in \{c\},$$

the extremal length $\lambda\{c\}$ with respect to $\{c\}$ is defined by

$$\lambda\{c\}^{-1} = \inf_{\rho} \overline{\iint_R} \rho^2(z)\, dx\,dy, \text{ where } z=x+iy \text{ is a uniformizer.}$$

Now we consider the system of curves $\{C\}\subset R-R_0$ (R_0 is an image of z-circle) such that each $C\in\{C\}$ consists of a finite number of disjoint *analytic* Jordan closed curves and C is homologous to ∂R_0 (the boundary of R_0). Then we can prove

PROPOSITION 1. *R is of parabolic type if and only if $\lambda\{C\}=0$.*

2. Let $\{\gamma\}$ be a subset of $\{C\}$ which contains an infinite number of curves of $\{C\}$ tending to the ideal boundary $\mathfrak{J}$ of R. Then we can prove the property which plays a fundamental role in the following.

PROPOSITION 2. *Suppose that φ_1 and φ_2 are any two non-negative covariants which are square integrable over $R-K$ (K is a compact domain with analytic boundaries). If $\lambda\{\gamma\}=0$, then there exists a sequence of curves $\gamma_n\in\{\gamma\}$ $(\gamma_n\cap K=\phi)$ tending to $\mathfrak{J}$ such that*

$$\underline{\int_{\gamma_n}} \varphi_1\,|dz|\, \underline{\int_{\gamma_n}} \varphi_2\,|dz| \to 0 \quad \textit{for } n\to\infty.$$

3. We take account of two subsets $\{\Gamma\}$, $\{L\}_E$ of $\{C\}$ as $\{\gamma\}$.

(I) $\{\Gamma\}$: $\{\Gamma\}$ denotes the set of curves $\Gamma\in\{C\}$ such that in the decomposition of Γ into its components each component divides R into two disjoint parts.

1) Kusunoki, Y.,: On Riemann's periods relations on open Riemann surfaces, Mem. Coll. Sci., Univ. Kyoto, Ser. A, Math., **30**, No. 1 (shortly appear).