93. On Solutions of a Partially Differential Equation with a Parameter

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(Comm. by K. KUNUGI, M.J.A., June 12, 1956)

Let $P\left(\frac{\partial}{\partial x}, \lambda\right)$ be a polynomial of derivations $\sum_{|p|\leq m} a_p(\lambda)\frac{\partial^p}{\partial x^p}$ defined in R^n and with a parameter λ, where $a_p(\lambda)$ are complex valued continuous functions on a separable and locally compact space Λ and where the degrees of polynomials $P(\xi, \lambda)$ are independent of $\lambda \in \Lambda$. Then we consider the existence of distribution solution, which is continuous with respect to $\lambda \in \Lambda$, of the partially differential equation

$$P\left(\frac{\partial}{\partial x}, \lambda\right)S_x(\lambda)=T_x(\lambda), \tag{1}$$

where $T_x(\lambda)$ is a given continuous function on Λ into a distribution space.

In the special case in this direction where Λ consists of a point, many interesting results are obtained by B. Malgrange, L. Hörmander and L. Ehrenpreis.[1] Furthermore recently F. Tréves[2] considered the case where $T_x(\lambda)=\delta$. Here we prove more general theorems applying considerations of these author's.

Theorem 1. *For any continuous function $T_x(\lambda)$ on Λ into $\mathfrak{D}'_x$ there is a solution $S_x(\lambda)$ of the equation (1) where $S_x(\lambda)$ is a continuous function on Λ into $\mathfrak{D}'_x$ and where $S_x(\lambda)=0$ whenever $T_x(\lambda)=0$.*

Theorem 2. *Under the same assumption of Theorem 1, if $S_x(\lambda)$ is a continuous solution such that*

$$P\left(\frac{\partial}{\partial x}, \lambda\right)S_x(\lambda)=T_x(\lambda) \qquad \text{for } \lambda \in \Lambda_0 \tag{2}$$

where Λ_0 is a closed subspace of Λ, then there is a continuous solution $S'_x(\lambda)$ of (1) defined over Λ such that

$$S'_x(\lambda)=S(\lambda) \qquad \textit{for } \lambda \in \Lambda_0. \tag{3}$$

Furthermore we may replace $\mathfrak{D}'_x$ by $\mathcal{E}_x$, that is, we can prove the following

1) B. Malgrange: Equations aux dérivées partielles à coefficients constants. I, II, C. R. Acad. Sci., Paris, **237** (1953), **238** (1954). L. Hörmander: On the theory of general partial differential operators, Acta Math., **94** (1955). L. Ehrenpreis: The division problem for distributions, Proc. Nat. Acad. Sci., **41** (1955).

2) F. Tréves: Solution élémentaire d'équations aux dérivées partielles dépendant d'un paramètre, C. R. Acad. Sci., Paris, **242** (1956).