89. On Closed Mappings. II

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The present note is a continuation of our previous paper on the closed mappings.[1] Let S and E be T_1-spaces. A mapping from S onto E is said to be closed if the image of every closed subset of S is closed in E. Recently it has been shown that several topological properties are invariant under a closed continuous mapping under some restrictions.[2]

In this note, we will prove the invariance of other topological properties under a closed continuous mapping and under the inverse mapping of it, under some restrictions.

1. Let us recall some definitions in the following. The space S is called paracompact (point-wise paracompact) if every open covering of S has an open locally finite (point-finite) refinement and countably paracompact if every countable open covering has an open locally finite refinement. The space S is said to have the star-finite property if every open covering of S has an open star-finite refinement. By an S-space, we mean a normal space with the star-finite property according to E. G. Begle.[3]

Theorem 1. *Let f be a closed continuous mapping from a normal space S onto a normal space E. If the inverse image $f^{-1}(p)$ is compact for every point p of E, then the countable paracompactness is invariant under f.*

Proof. Since f is a closed continuous mapping, the image space E is normal by a theorem of G. T. Whyburn.[4] Let $\{F_i\}$ be a decreasing sequence of closed sets in E with vacuous intersection. Then $\{f^{-1}(F_i)\}$ is a decreasing sequence of closed sets in S with vacuous intersection since f is continuous. Since S is countably paracompact and normal, there exists a sequence $\{G_i\}$ of open sets such that $\bigcap_{i=1}^{\infty} G_i=\phi$ and $f^{-1}(F_i)\subset G_i$ $(i=1, 2,\cdots)$.[5] Since f is closed and continu-

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2) G. T. Whyburn: Open and closed mappings, Duke Math. Jour., **17**, 69–74 (1950). A. V. Martin: Decompositions and quasi-compact mappings, Duke Math. Jour., **21**, 463–469 (1954). V. K. Balachandran: A mapping theorem for metric spaces, Duke Math. Jour., **22**, 461–464 (1955). K. Morita and S. Hanai: Closed mappings and metric spaces, Proc. Japan Acad., **32**, 10–14 (1956).

3) E. G. Begle: A note on S-spaces, Bull. Amer. Math. Soc., **55**, 577–579 (1949).

4) G. T. Whyburn: Loc. cit.

5) C. H. Dowker: On countably paracompact spaces, Canadian Jour. Math., **3**, 219–224 (1951).