87. *Some Strong Summability of Fourier Series. II*

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1. Let $u(x)$ be integrable L^p $(p>1)$, periodic with period 2π and let $s_n(x)$ be the nth partial sum of its Fourier series. Then S. Izumi [5] proved that *if* $p\geqq k>1$, $\varepsilon>0$ *and*

$$\left(\int_{-\pi}^{\pi}|u(x+t)-u(x)|^p dx\right)^{1/p}\leqq K\{t^{1/k}(\log 1/t)^{-(1+\varepsilon)/k}\},$$

then the series

$$\sum_{n=1}^{\infty}|s_n(x)-u(x)|^k \tag{1.1}$$

converges for almost all x. Concerning the convergence of the series (1.1), S. Izumi [4] and the author [6, 7] have gotten some related results.

In this paper, we shall prove more general theorems concerning the series (1.1), replacing the partial sum $s_n(x)$ by the Cesàro mean $\sigma_n^\delta(x)$.

2. Suppose that

$$f(z)=\sum_{n=0}^{\infty}c_n z^n,\quad (z=re^{i\theta}),$$

is an analytic function of z, regular for $|z|=r<1$ and its boundary function is $f(e^{i\theta})$. Then we say that[1] $f(z)$ belongs to the "complex" class Lip (α,β,p) if it satisfies

$$M_p(r,f')=\left(\frac{1}{2\pi}\int_{-\pi}^{\pi}|f'(re^{i\theta})|^p d\theta\right)^{1/p}=O\left\{(1-r)^{-1+\alpha}\left(\log\frac{1}{1-r}\right)^{-\beta}\right\}.$$

Throughout this paper we use the following notation:

$$\sigma_n^0(\theta)=s_n(\theta)=\sum_{\nu=0}^{n}c_\nu e^{i\nu\theta},$$

$$\sigma_n^\delta(\theta)=\frac{1}{A_n^\delta}\sum_{\nu=0}^{n}A_{n-\nu}^{\delta-1}s_\nu(\theta),\quad \text{for } \delta>-1,$$

$$t_n(\theta)=nc_ne^{in\theta},$$

$$\tau_n^\delta(\theta)=\frac{1}{A_n^\delta}\sum_{\nu=0}^{n}A_{n-\nu}^{\delta-1}t_\nu(\theta),\quad \delta>0,$$

where

$$A_n^\delta=\binom{n+\delta}{n}\sim\frac{n^\delta}{\Gamma(\delta+1)}.$$

Then we have $\tau_n^\delta(\theta)=n\{\sigma_n^\delta(\theta)-\sigma_{n-1}^\delta(\theta)\}=\delta\{\sigma_n^{\delta-1}(\theta)-\sigma_n^\delta(\theta)\}$.

Our results may now be stated as follows:

1) Cf. Hardy-Littlewood [2].