85. *Polarized Varieties, the Fields of Moduli and Generalized Kummer Varieties of Abelian Varieties*[1)]

By Teruhisa MATSUSAKA
Ochanomizu University, Tokyo, and University of Chicago
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1. Let V be a complete variety non-singular in co-dimension 1 and F be an algebraic family of positive V-divisors. We shall say that F is a *total* family if for every divisor Z algebraically equivalent to 0 on V, there is a divisor X in F such that

$$Z \sim X - X_0$$

with a fixed X_0 in F. F is called a *maximal* family, if there is no algebraic family containing F as a sub-family. In particular F is called a *complete* family if every positive divisor which is algebraically equivalent to a divisor in F is already contained in F and if every divisor in F determines the complete linear system of the same dimension. A linear system on V is called *ample* if it determines a projective imbedding of V, i.e., an everywhere biregular birational transformation of V into a projective space. When a linear system is ample, it is clear that the complete linear system determined by it is ample. Let X be a V-divisor. We shall say that X is *linearly effective* if the complete linear system determined by X is ample. We shall say that X is *algebraically effective*, if every divisor which is algebraically equivalent to X is linearly effective. Finally we shall say that X is *numerically effective*, if every divisor Y such that mY is algebraically equivalent to mX for a convenient integer m, is linearly effective.

When V is a projective variety, there is a finite number of maximal algebraic family containing the given divisor X, and in fact, the set of positive V-divisors of the given degree forms a finite number of maximal families (Chow-v.d. Waerden [2]). Also in this case, there is a total family on V and when X is any divisor on V and C is a hyperplane section of V, there is a total family which is a set of positive divisors algebraically equivalent to $X+mC$ for large m (Matsusaka [3, 4]). In this paper, we need the following theorem on maximal families on non-singular projective varieties.

Theorem 1. Let V be a non-singular variety in a projective space

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