99. *Notes on Topological Spaces. V. On Structure Spaces of Semiring*

By Kiyoshi ISÉKI
Kobe University

(Comm. by K. KUNUGI, M.J.A., July 12, 1956)

The structure space of maximal ideals in a normed ring has been discussed by I. Gelfand and G. Silov [2]. Recently, the structure space of maximal ideals in a semiring has been considered by W. Slowikowski and W. Zawadowski [4] and the theory has generalized by the present author and Y. Miyanaga [3]. E. A. Behrens [1] has considered the relation of structure spaces formed by three special classes of ideals of a naring.

In this Note, we shall consider the *structure space* $\mathfrak{P}$ of all prime ideals of a semiring A and the relation of $\mathfrak{P}$ and the structure space $\mathfrak{M}$ of all maximal ideals of A. Throughout the paper, we shall treat a commutative semiring A with a zero 0 and a unit 1. (For detail of the definition, see K. Iséki and Y. Miyanaga [3].)

An ideal P of A is *prime* if and only if $ab \in P$ implies $a \in P$ or $b \in P$. Since A has a unit 1, any maximal ideal is prime, therefore $\mathfrak{P} \supseteq \mathfrak{M}$.

To introduce a topology γ on $\mathfrak{P}$, we shall take $\gamma_x = \{P \mid x \bar{\in} P, P \in \mathfrak{P}\}$ for every x of A as an open base of $\mathfrak{P}$. Then we have the following

Theorem 1. Let $\mathfrak{A}$ be a subset of $\mathfrak{P}$, then

$$\bar{\mathfrak{A}} = \{P' \mid \bigcap_{P \in \mathfrak{A}} P \subset P' \text{ and } P' \in \mathfrak{P}\},$$

where $\bar{\mathfrak{A}}$ is the closure of $\mathfrak{A}$ by the topology γ.

Proof. To prove that the $\bar{\mathfrak{A}}$ contains $\{P' \mid \bigcap_{P \in \mathfrak{A}} P \subset P', P' \in \mathfrak{P}\}$, let $P' \in \{P' \mid \bigcap_{P \in \mathfrak{A}} P \subset P', P' \in \mathfrak{A}\}$, and let γ_x be a neighbourhood of P', then $x \bar{\in} P'$, and we have $x \bar{\in} \bigcap_{P \in \mathfrak{A}} P$. Therefore, there is a prime ideal $P \in \mathfrak{A}$ such that P does not contain x and $\gamma_x \ni P$. This shows $P \in \bar{\mathfrak{A}}$.

If a prime ideal P' is not in $\{P' \mid \bigcap_{P \in \mathfrak{A}} P \subset P', P' \in \mathfrak{A}\}$, then $\bigcap_{P \in \mathfrak{A}} P - P'$ is not empty. Hence, for $x \in \bigcap_{P \in \mathfrak{A}} P - P'$, we have $x \in P, P \in \mathfrak{A}$ and $x \bar{\in} P'$. This shows $\gamma_x \bar{\ni} P$, $P \in \mathfrak{A}$ and $\gamma_x \bar{\ni} P'$. Therefore $\gamma_x \frown \mathfrak{A} = 0$ and hence $P' \bar{\in} \bar{\mathfrak{A}}$. The proof is complete.

A similar argument for $\mathfrak{M}$ relative to Γ-topology implies the following

Proposition. Let $\mathfrak{A}$ be a subset of $\mathfrak{M}$, then