131. On Natural Systems of Some Spaces

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In this note we shall give a brief account about the properties of natural systems and Postnikov's complexes. We state here only the results without proofs.[1] Full details will appear in a forthcoming Journal of the Faculty of Science, Niigata University.

§1. Let X be an arcwise-connected, simply-connected topological space. We shall denote the i-th homotopy group $\pi_i(X, x_0)$ and the natural system of X by π_i and (π_i, k_i) respectively. Let K_i and e^r be the cell-complex of (π_i, k_i) and the unique r-cell of $K_1=K(\pi_1)$ respectively.

Let $\hat{X}$ be the space of loops on X with x_0 as the end point. Hereafter each notation covered by ^ denotes the notation concerned with $\hat{X}$. In particular, $\hat{e}^r$ is the r-dimensional matrix (d_{ij}) where d_{ij} is the unit element of $\hat{\pi}_1$ for each i and j.

In the first place we must note that the following theorem can be proved:

Theorem 1. *$\hat{\pi}_1$ operates trivially on $\hat{\pi}_n$ $(n\geqq 2)$.*

We now define $\rho_{r+1}: \Delta^{r+1}\to\Delta^r\times I$ by

$$\rho_{r+1}(y_1, y_2,\cdots, y_{r+1})=\begin{cases}(ly_1, ly_2,\cdots, ly_{r+1}), & (y_1+y_2+\cdots+y_r\geqq y_{r+1}),\\ (my_1, my_2,\cdots, my_{r+1}), & (y_1+y_2+\cdots+y_r\leqq y_{r+1}),\end{cases}$$

where $l=\dfrac{y_1+y_2+\cdots+y_{r+1}}{y_1+y_2+\cdots+y_r}$, $m=\dfrac{y_1+y_2+\cdots+y_{r+1}}{y_{r+1}}$ and

$$\Delta^{r+1}=\{(y_1, y_2,\cdots, y_{r+1}):\ 0\leqq y_i\leqq 1\ (i=1,2,\cdots, r+1),\ 0\leqq y_1+y_2+\cdots+y_{r+1}\leqq 1\}$$

is an $(r+1)$-dimensional Euclidean simplex and Δ^r is the r-face $\Delta^{r+1(r+1)}$ of Δ^{r+1} contained in the hyperplane $y_{r+1}=0$.

Let $\hat{T}^r:\Delta^r\to\hat{X}$ be an r-dimensional singular simplex of $\hat{X}$ and define $\xi_{r+1}:\Delta^r\times I\to X$ by $\xi_{r+1}(P,s)=\hat{T}^r(P)(s)$ where $P\in\Delta^r$ and $s\in I=[0,1]$ is the parameter of loops. Define τ by $\tau\hat{T}^r=\xi_{r+1}\circ\rho_{r+1}:\Delta^{r+1}\to X$. We use the same notation τ for the induced map: $[\hat{T}^r]\to[T^{r+1}]$ subject to the condition that $[\hat{T}^r]$ is an element of $\hat{\pi}_r$, where T^{r+1} is an $(r+1)$-dimensional singular simplex of X. It is easily seen that:

1) τ is an isomorphism of $\hat{\pi}_r$ onto π_{r+1},

1) In this note we quote the notations and definitions from the following report without essential modifications: P. J. Hilton: Report on three papers by M. M. Postnikov (1952).