# 126. On Decomposition Spaces of Locally Compact Spaces

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**1. Introduction.** As is well known, any decomposition space[1] of a compact Hausdorff space is normal if it is Hausdorff. The following theorem is a generalization of this fact:[2]

**Theorem 1.** *Let a Hausdorff space $Y$ be a decomposition space of a Hausdorff space $X$. If $X$ is locally compact and has the Lindelöf property, then $Y$ is paracompact and normal.*

Theorem 1 fails to be true if we replace the condition "has the Lindelöf property" by "is paracompact". This is seen from Theorem 2 below; further it will be shown in 5 below that a Hausdorff space which is obtained as a decomposition space of a locally compact, paracompact, Hausdorff space is not always regular.

**Theorem 2.** *A Hausdorff space $X$ is obtained as the image of a locally compact, paracompact, Hausdorff space under an open continuous mapping if and only if $X$ is locally compact.*

In [1, p. 70] P. Alexandroff and H. Hopf have stated that the existence of a regular, non-normal, Hausdorff space which is a decomposition space of a normal Hausdorff space remains unknown to them. Our Theorem 2 assures the existence of such a decomposition space[3] and settles this question, since there exists a non-normal, locally compact Hausdorff space. However, the following theorem will give a stronger result.

**Theorem 3.** *A Hausdorff space $X$ is obtained as the image of a locally compact metric space under an open continuous mapping if and only if $X$ is locally compact and locally metrizable.*

In the Euclidean plane, let $E$ be the union of the line $x=0$ and the points $a_{nk}=\left(\frac{1}{n}, \frac{k}{n^2}\right)$, $n=1,2,\cdots$; $k=0, \pm 1, \pm 2, \cdots$ . If the sets $T_n(y)$, $n=1,2,\cdots$; $-\infty<y<\infty$ and one-point sets $\{a_{nk}\}$, $n=1,2,\cdots$;

1) "Decomposition space" = "Zerlegungsraum" in the sense of [1, p. 63].

2) There exists a non-regular Hausdorff space which is the image, under an open continuous mapping, of a metric space which is a countable sum of compact sets; cf. [1, p. 70, Beispiel 2], where in line 15 from the bottom "$m=2,3,\cdots$" should be replaced by "$m=n, n+1,\cdots$".

3) If $g$ is an open (or closed) continuous mapping of a $T_1$-space $Z$ onto a $T_1$-space $X$, then $X$ is homeomorphic to a decomposition space of $Z$ associated with the decomposition $\{g^{-1}(x) \mid x \in X\}$ (cf. [1, p. 65] and [8]).