152. *Note on Free Algebraic Systems*

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In his paper,[1] K. Shoda has defined only the free A-algebraic systems, when he has discussed the free algebraic systems. However, in this note, we shall define free algebraic systems more generally. And we shall show a generalization of the Shoda's fundamental theorem[2] (Theorems 1, 2 and 3), and a necessary and sufficient condition for the existence of the free algebraic system with an arbitrary set of relations (Theorem 3). Finally, we shall show a characterization of the algebraic systems defined by only a set of relations, i.e. the A-algebraic systems satisfying a set of relations (Theorem 4).

Throughout this note, the system V of single-valued compositions will be fixed. Let E be a set of generators, then the absolutely free algebraic system $F(E,\phi)$[3] is defined. And let P be a family of postulates with respect to V and E, then P-algebraic systems generated by E are defined as residue class systems of $F(E,\phi)$ satisfying P. And (E,P) denotes the set of all P-algebraic systems generated by E. Moreover, let R be a set of relations (identities) in $F(E,\phi)$, then the P-algebraic systems satisfying R generated by E are defined. And (E,P,R) denotes the set of all such P-algebraic systems.

An algebraic system $\mathfrak{F}$ is called a free P-algebraic system with a set R generated by E, or a free algebraic system belonging to (E,P,R), when $\mathfrak{F}$ is contained in (E,P,R) and every algebraic system in (E,P,R) is a residue class system of $\mathfrak{F}$. And we denote it by $F(E,P,R)$.

Theorem 1. *If an algebraic system $\mathfrak{A}$ is contained in (E,P,R), then there exists a set S of relations satisfying $\mathfrak{A}=F(E,P,S)$ and $S\supseteq R$.*

Proof. Let $\mathfrak{A}\in(E,P,R)$, then it is clear that $\mathfrak{A}\in(E,\phi,R)$.[4] Hence there exists a set S of relations satisfying $\mathfrak{A}=F(E,\phi,S)$ and $S\supseteq R$ by

1) K. Shoda: Allgemeine Algebra, Osaka Math. J., **1** (1949).

2) Using our notations, we can show the Shoda's fundamental theorem for the free algebraic systems as follows: Let P be a family of composition-identities. Then i) there exists a free algebraic system $F(E,P,R)$ for every set R of relations, ii) if an algebraic system $\mathfrak{A}$ is contained in (E,P,R), then there exists a set S of relations satisfying $\mathfrak{A}=F(E,P,S)$ and $S\supseteq R$, and iii) if $R\subseteq S$, then $F(E,P,S)$ is a residue class system of $F(E,P,R)$.

3) In his paper 1), K. Shoda has denoted by $O(E)$ the absolutely free algebraic system.

4) ϕ denotes the empty set.