161. *On Linear Hyperbolic System of Partial Differential Equations in the Whole Space*

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J. Leray gave the existence theorem for Cauchy problem of very general hyperbolic differential equations, but his work is not so simple to be followed easily in detail [1]. K. Friedrichs reduced the hyperbolic differential equations of second order to a symmetric system of differential equations of first order and solved the Cauchy problem in a lens-shaped domain [2]. He proved the existence of extended solutions by Hilbert space method, but showed the differentiability of solutions by somewhat complicated calculations of difference equations. P. Lax represented an elegant method which offers both the existence and the differentiability of solutions at once for the symmetric hyperbolic system of equations [3]. He reduced the problem to the case that all functions are periodic in every independent variables, but it seems me not so adequate to obtain solutions in the whole space.

The object of this paper is to give an existence theorem for Cauchy problem in the whole space and in such an abstract form that it may cover a general class of hyperbolic systems, even parabolic equations. In this note we state only the main results and related lemmas without proof. All details will be published later. Our main idea owes to Lax. Our investigation is also much stimulated by the conversations with Dr. T. Shirota and Prof. K. Yosida [4]. Especially I owe to Shirota the generalization of the operator Λ such as (3), to make it suitable to a wide class of equations [5].

1. Solutions without Initial Condition

Let $x=(x_1,\cdots,x_m)$ be a variable point of m-dimensional Euclidean space E^m, and $u=(u_1,\cdots,u_l)$ be l-dimensional vector, whose components are real valued functions of $x\in E^m: u=u(x)$. By (u, v) we denote the inner product

$$(u, v)=\int_{E^m}\sum_{i=1}^{l} u_i v_i\, dx$$

and $\|u\|$ denotes the norm of $u: \|u\|=\sqrt{(u, u)}$. With this norm we get a real Hilbert space H_0:

$$H_0=\{u;\ \|u\|<\infty\}.$$

Let Λ be a self-adjoint linear differential operator such that