24. *On the Cut Operation in Gentzen Calculi. II*

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The object of this Note is to give an exact form of Theorem 1 in my Note [1]. Theorem 1 is incorrect and its proof is not sufficient. We follow the terminologies and notations in my Note [1] in the sequel.

Theorem 1. The cut rule in LK-system is replaced by

$$(1) \qquad \frac{\Gamma \to \mathfrak{A} \supset \mathfrak{B}, \Delta \quad \Pi, \Gamma \to \mathfrak{A}}{\Gamma, \Pi \to \Delta, \mathfrak{B}}$$

and

$$(2) \qquad \frac{\to \mathfrak{A} \quad \mathfrak{A} \to}{\to}$$

Proof. In my Note [1], we proved that the cut rule implies (1), and (2) follows from the cut rule immediately. To prove that (1) and (2) imply the cut rule:

$$(3) \qquad \frac{\Gamma \to \Delta, \mathfrak{A} \quad \mathfrak{A}, \Pi \to \Lambda}{\Gamma, \Pi \to \Delta, \Lambda}$$

If Λ is not empty, there is a proposition $\mathfrak{B}$ in Λ. Then we have the following proof.

$$\dfrac{\dfrac{\dfrac{\dfrac{\mathfrak{A}, \Pi \to \Lambda}{\mathfrak{A}, \Pi \to \Lambda_{\mathfrak{B}}, \mathfrak{B}}}{\Pi \to \Lambda_{\mathfrak{B}}, \mathfrak{A} \supset \mathfrak{B}} \quad \Gamma \to \mathfrak{A}, \Delta}{\Pi, \Gamma \to \Lambda_{\mathfrak{B}}, \Delta, \mathfrak{B}}}{\Pi, \Gamma \to \Delta, \Lambda}$$

In (3), if Λ is empty, we have

$$(4) \qquad \frac{\Gamma \to \Delta, \mathfrak{A} \quad \mathfrak{A}, \Pi \to}{\Gamma, \Pi \to \Delta}$$

If Π is not empty, Π contains a proposition $\mathfrak{B}$, and then we have,

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\mathfrak{A}, \Pi \to}{\mathfrak{A}, \mathfrak{B}, \Pi_{\mathfrak{B}} \to}}{\mathfrak{A}, \Pi_{\mathfrak{B}} \to \neg\mathfrak{B}}}{\Pi_{\mathfrak{B}} \to \mathfrak{A} \supset \neg\mathfrak{B}} \quad \Gamma \to \Delta, \mathfrak{A}}{\Gamma, \Pi_{\mathfrak{B}} \to \Delta, \neg\mathfrak{B}}}{\Gamma, \Pi_{\mathfrak{B}}, \mathfrak{B} \to \Delta}}{\Gamma, \Pi \to \Delta}$$

This shows (4). If Π is empty, cut rule is

$$\frac{\Gamma \to \Delta, \mathfrak{A} \quad \mathfrak{A} \to}{\Gamma, \to \Delta}$$