23. *Divergent Integrals as Viewed from the Theory of Functional Analysis. I*

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§ 1. *Introduction.* Let the complex valued function $f(z, \lambda)$ be defined for a real number λ, $(a \leq \lambda \leq b)$ and for a complex number z on a domain D and also on another domain D_1. We assume that the integral $\int_a^b f(\lambda, z)d\lambda$ converges for $z \in D$ and diverges for $z \in D_1$. We denote the convergent integral for $z \in D$ by $f(z)$.

Now in many branches of analysis, it is often necessary or convenient to use a function $f^*(z)$ on $z \in D_1$ which in some senses corresponds to $f(z)$ on $z \in D$. For example, (i) when $f(z)$ is analytic on D, the analytic extension in D_1 of $f(z)$ may be taken as $f^*(z)$, (ii) when $f(z, x)$ is a solution for $z \in D$ of some differential or integral equations which contain z as a parameter, the solution for $z \in D_1$ may be taken as $f^*(z)$. To derive the functions $f^*(z)$ in such cases, we have many classical methods: changings of the contour of integral, or various methods of summation, or other limiting processes.

In this and the following papers we always view such divergent integrals from the theory of functional analysis. We construct a functional space Φ whose elements for example are the functions $\varphi(\sigma, \tau)$ defined on D_1 or on both D and D_1 having suitable properties there (here σ and τ are respectively the real part and the imaginary part of z).

Now the mapping $\lambda \to f(\lambda, z)$ defines a mapping from $a \leq \lambda \leq b$ to Φ', where Φ' is a dual space of Φ. We regard the divergent integral $\int_a^b f(\lambda, z)d\lambda$ for $z \in D_1$ as the integral in Φ'. If it converges weakly or strongly we can examine whether the functional $f^*(z)$ defines a function or not and we can investigate its properties also.

We consider particularly about the following type of divergent integrals (1), though, of course, the similar method will be able to be adopted for other sorts of kernels, dimensions of λ or intervals of the integration by selecting suitable functional spaces Φ.

This type of integrals relates to some of the most important parts of classical analysis, i.e. the Laplace transform, the power series development, the analytic continuation, some sorts of approximations, and some differential or integral equations (see § 2).

In this paper we show some sufficient conditions for these kernels,