20. *Analytic Functions in the Neighbourhood of the Ideal Boundary*

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Let R be a Riemann surface with null-boundary and let $\{R_n\}$ be its exhaustion with compact relative boundary. We proved the following

Theorem 1.[1] *Let R' be a subsurface of R with compact relative boundary. Let $f(z)$ be a bounded analytic function on R'. Then $f(z)$ has a limit as z tends to an ideal boundary component of R'.*

We extend this theorem to more general class of Riemann surfaces. Let R be a Riemann surface with positive boundary and let R' be a subsurface of R with compact relative boundary Γ. We introduce two classes of Riemann surfaces.

There exists no non-constant one valued bounded (Dirichlet bounded) harmonic function $U(z)$ on R' such that $U(z)=0$ on Γ, the period of the conjugate function of $U(z)$ vanishes along every dividing cut of R. We say $R\in O'_{AB}$ and $\in O'_{AD}$ respectively. O'_{AB} and O'_{AD} are the extension of the classes of O_{AB} and O_{AD} of the Riemann surface of finite genus. We see easily that the property $\in O'_{AB}$ ($\in O'_{AD}$) is the one depending only on the ideal boundary.

Theorem 2. *Suppose a bounded (Dirichlet bounded) analytic function on $R'\in O'_{AB}(O'_{AD})$. Then $f(z)$ has a limit as z tends to a boundary component of R'.*

To prove Theorem 2 we make some preparations.

Let R be a Riemann surface with positive boundary and let $\{R_n\}$ $(n=0,1,2,\cdots)$ be its exhaustion with compact relative boundary $\{\partial R_n\}$. Let $N(z,p): p\in R$ be a positive harmonic function in $R-R_0$ such that $N(z,p)=0$ on ∂R_0, $N(z,p)$ has a logarithmic singularity at p and $N(z,p)$ has the minimal $*$-Dirichlet integral.[2] Let $\{p_i\}$ be a sequence tending to the ideal boundary of R such that $\{N(z,p_i)\}$ converges uniformly in every compact domain of R. We say that $\{p_i\}$ is a fundamental sequence determining an ideal boundary point and we make $\lim_{i=\infty} N(z,p_i)$ correspond to this ideal boundary point. Denote by B the ideal boundary point. The distance between points p_1 and p_2 of $R-R_0+B$ is defined by

1) Z. Kuramochi: Potential theory and its applications, I, Osaka Math., **3** (1951).
2) Z. Kuramochi: Mass distributions on the ideal boundaries of abstract Riemann surfaces, II, Osaka Math., **8** (1956).