# 19. On $LC^n$ Metric Spaces

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1. Introduction. A topological space $X$ is called an $LC^n$ space [7, p. 79] if for any point $x$ of $X$ and any neighborhood $U$ of $x$ there exists a neighborhood $V$ of $x$ such that any continuous mapping $g:s^i\to V$, $i=0,1,\cdots,n$, has an extension $\tilde{g}:E^{i+1}\to U$, where $S^i$ is an $i$-dimensional sphere and $E^{i+1}$ is an $(i+1)$-dimensional element with the boundary $S^i$. A topological space $X$ is called a $C^n$ space [7, p. 78] if any continuous mapping $g:s^i\to X$, $i=0,1,\cdots,n$, has an extension $\tilde{g}:E^{i+1}\to X$. A topological space $X$ is called an $n$-ES (resp. $n$-NES) [6] for metric spaces if, whenever $Y$ is a metric space, $B$ is a closed subset of $Y$ such that $\dim(Y-B)\leqq n$[1)] and $g$ is any continuous mapping $B$ to $X$, there exists an extension $\tilde{g}$ of $g$ from $Y$ (resp. some neighborhood of $B$ in $Y$) to $X$. A metric space $X$ is called an $n$-AR (resp. $n$-ANR) for metric spaces if, whenever $Y$ is a metric space in which $X$ is closed and $\dim(Y-X)\leqq n$,[1)] $X$ is a retract [1] of $Y$ (resp. some neighborhood of $X$ in $Y$).

In this paper, we shall prove the following theorems concerning $LC^n$ spaces:

Theorem 1. An $n$-dimensional metric space[1)] is an ANR for metric spaces if and only if it is an $LC^n$ space.

Theorem 2. An $n$-dimensional $LC^n$ metric space $X$ is an $n$-ES for metric spaces if and only if $\pi_i(X)=0$, $i=0,1,\cdots,n-1$, and $\pi_n(X)$ is 0 or the weak product of infinite cyclic groups, where $\pi_j(X)$ is the $j$-dimensional homotopy group of $X$.

Theorem 3. For a metric space $X$ the following conditions are equivalent:

i) $X$ is an $LC^n$ space.

ii) $X$ is an $(n+1)$-NES for metric spaces.

iii) $X$ is an $(n+1)$-ANR for metric spaces.

Theorem 4. If $X$ is an $LC^n$ metric space, for each integer $i=0$, $1,\cdots,n$, the following conditions are equivalent:

i) $X$ is a $C^i$ space.

ii) $X$ is an $(i+1)$-ES for metric spaces.

iii) $X$ is an $(i+1)$-AR for metric spaces.

S. Lefschetz [7] proved Theorem 1 in case $X$ is a compact

1) In this paper, we understand by "dimension" the covering dimension. (For example, see [8, p. 350].)