18. Fourier Series. XIII. Transformation of Fourier Series

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Let $\Lambda=(\lambda_{\nu n})$ $(\nu, n=0,1,2,\cdots)$ be an infinite matrix whose elements are real numbers and let $f(x)$ be an integrable function periodic with period 2π, and its Fourier series be

$$(1) \qquad f(x)\sim\frac{1}{2}a_0+\sum_{\nu=1}^{\infty}(a_\nu \cos \nu x+b_\nu \sin \nu x).$$

The Fourier series (1) is said to be Λ-summable to $\alpha(x)$, if the series

$$(2) \qquad \alpha_n(x)=\frac{1}{2}a_0\lambda_{0n}+\sum_{\nu=1}^{\infty}\lambda_{\nu n}(a_\nu \cos \nu x+b_\nu \sin \nu x)$$

converges for all n and

$$(3) \qquad \lim_{n\to\infty}\alpha_n(x)=\alpha(x) \quad \text{exists.}$$

If the convergence of both (2) and (3) is uniform in x, it is said to be uniformly Λ-summable to $\alpha(x)$.

Concerning the Λ-summability of Fourier series of all continuous functions, J. Karamata [1] has established the following

Theorem. *A necessary and sufficient condition that the Fourier series of all continuous functions be uniformly Λ-summable to $f(x)$ is that*

1) $$\lim_{n\to\infty}\lambda_{\nu n}=1 \quad \textit{for every } \nu,$$

2) $$\int_0^\pi |K_{mn}(t)|\,dt\leqq M_n \qquad (n=0,1,2,\cdots),$$

where $K_{mn}(t)=\frac{1}{2}\lambda_{0n}+\sum_{\nu=1}^{m}\lambda_{\nu n}\cos \nu t$ *and M_n is independent of m, and*

3) $$\int_0^\pi |d\overline{K}_n(t)|=O(1) \quad \textit{for all } n,$$

where

$$\overline{K}_n(x)=\lim_{m\to\infty}\int_0^x K_{mn}(t)dt.$$

In this note, we shall prove L_p-analogues $(p\geqq 1)$ of this theorem. For the proof we use the following theorems [2]:

Theorem A. *A necessary and sufficient condition that $\{\lambda_n\}$ be a sequence of uniform convergence factors of Fourier series of all functions belonging to L_p $(p>1)$, is that*

$$\int_{-\pi}^{\pi}|K_n(t)|^q dt=O(1) \qquad (n\to\infty),$$