16. *Certain Subgroup of the Idèle Group*

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Let k be an algebraic number field of finite rank over the rational number field Q, I the group of idèles of k, P the group of principal idèles of k, C the idèle class group I/P, H' the maximal compact subgroup in the connected component H of the unit element of I, D' the natural image (isomorphic) of H' into C, and D the connected component of the unit element of C. Clearly $D\supset D'$, and D/D' is, as shown by Weil in his article [5], an infinitely and uniquely divisible group.[1] Combining it with Grunwald's lemma corrected by Wang and Hasse,[2] we shall prove in the present article the following

Theorem. Let J be the subgroup in I consisting of all of such idèles each of which has 1 as its component at every prime divisor of k except a nulset (with reference to Kronecker density) of finite prime divisors of k. Then, the natural homomorphism ν of J into C/D is an isomorphism.

We prepare two lemmas. Let n be a natural number, ζ_{2^n} a primitive 2^n-th root of 1, $L_n=Q(\zeta_{2^n})\frown k$. Clearly, there exists a natural number N' such that for every n greater than N', $L_n=L_{N'}$. Let $N=N'+3$. Then, it holds the following

Lemma 1. Let l be a natural prime number and n a natural number greater than M_l, where $M_l=1$ for $l\neq 2$ and $M_l=N$ for $l=2$. Let α be a number in k such that α is l^n-th power residue at every prime divisor of k except a nulset (with reference to Kronecker density) of prime divisors of k. Then, α is l^{n-1}-th power of a number in k.

Proof. When $\alpha=0$, the lemma is trivial. Let α be a non zero number in k satisfying the condition of the lemma. Then, there exists a set T of finite prime divisors of k with 1 as its Kronecker density such that for each $p\in T$, α is l^n-th power of an element in the completion field k_p of k for p. So, α is l^n-th power of a number in $k(\zeta)$, where ζ is a primitive l^n-th root of 1. Then, α is, from Theorem 1 (Satz 1) in Hasse's article [3], l^n-th power of a number in k, if $l\neq 2$, and α is from the supposition for N and from Theorem 2 (Satz 2) in the above quoted article [3], l^{n-1}-th power of a number in k, even if $l=2$, and we obtain the lemma.

Lemma 2. Let p be a finite prime divisor of k, α a non zero

1) Cf. [1].

2) Cf. [2], [3], [4], esp. [3].