# 36. *A Remark on the Ranked Space. II*[1)]

By Tameharu SHIRAI

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1. We have proved the Baire's theorem with respect to the changed topology[2)] in the case that the system of neighbourhoods of a space does not satisfy the axiom (C) of F. Hausdorff, while there is an example[3)] of a ranked space which is complete with respect to the given topology but not with respect to the changed topology. Therefore, it is desirable to show the Baire's theorem with respect to the given topology. The purpose of this paper is to do this by modifing the definition of non-density in the case that the system of neighbourhoods does not satisfy the axiom (C).

*Definitions.* Let $R$ be a space having a system of neighbourhoods which satisfies the axiom (A) of F. Hausdorff, and denote by $\mathfrak{T}$ its topology. Excepting the two extreme cases,[4)] we shall modify the definition of the depth $\omega(R,p)$ of a point $p$. The monotone decreasing sequence of neighbourhoods

(1) $v_0(p_0)\supseteqq v_1(p_1)\supseteqq\cdots\supseteqq v_\alpha(p_\alpha)\supseteqq\cdots;\ 0\leq\alpha<\beta,$

will be called of *type* $\beta$, and the sequence (1) will be called *maximal* if there exists no neighbourhood $v(p)$ such that $v(p)\subseteqq\bigcap_{\alpha<\beta} v_\alpha(p_\alpha)$. The sequence of neighbourhoods of a fixed point $p$

(2) $v_0(p)\supseteqq v_1(p)\supseteqq\cdots\supseteqq v_\alpha(p)\supseteqq\cdots;\ \alpha<\beta,$

will be called *maximal at the point* $p$ if there exists no neighbourhood $v(p)$ of the point $p$ such that $v(p)\subseteqq\bigcap_{\alpha<\beta} v_\alpha(p)$. It is clear that the sequence (2), which is maximal at the point $p$, is not necessarily to be maximal in the general meaning. Let $v(p)$ be an arbitrary neighbourhood of $p$, and denote by $\omega(R,v(p))$ the minimum ordinal number of the types of maximal sequences having $v(p)$ as its first term, and we shall call $\omega(R,v(p))$ the *depth of* $v(p)$. Put into $\omega(R,p)=\inf_{v(p)}\omega(R,v(p))$ and we shall call $\omega(R,p)$ *the depth of the point* $p$. The depth $\omega(R)$[5)] of the space $R$ and the ranked space[6)] shall be defined by using the new $\omega(R,p)$.

It must be remarked that the depth defined by Prof. K. Kunugi

1) We shall hereafter translate "l'espace rangé" into "the *ranked* space".

2) T. Shirai: A remark on the ranged space, Proc. Japan Acad., **32** (1956).

3) See Example at the end of this paper.

4) K. Kunugi: Sur les espaces complets et régulièrement complets. I, Proc. Japan Acad., **30** (1954).

5) K. Kunugi: *Op. cit.*

6) K. Kunugi: *Op. cit.*