33. A Remark on Countably Compact Normal Space

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In [4], S. Kasahara, one of my colleagues, and I myself gave a new characterization of countably compact normal space. In this Note, we shall give a slight generalisation of Theorem 3 in [4]. Some writers ([2], [3], [5] and [6]) introduced the concepts of σ-discrete, σ-locally finite, and σ-star finite coverings of topological space, and they obtained the interesting results on some topological spaces. Let α be a family of open sets in a topological space S. α is said to be *discrete*, if every point of S has a neighbourhood which meets at most one member of α. α is said to be *point finite*, if every point of S is contained in only finite many members of α. α is said to be *star finite*, if every member of α meets only finite many members of α. α is said to be *locally finite*, if every point of S has a neighbourhood which meets only finite members of α.

An open covering α is called *σ-discrete* (*σ-point finite*, *σ-star finite* or *σ-locally finite*), if $\alpha=\bigcup_{i=1}^{\infty}\alpha_i$ such that each α_i is discrete (point finite, star finite or locally finite). Then we shall prove the following

Theorem 1. The following propositions of a normal space S are equivalent:

1) *S is countably compact.*
2) *Every σ-point finite open covering has a finite subcovering.*
3) *Every σ-locally finite open covering has a finite subcovering.*
4) *Every σ-star finite open covering has a finite subcovering.*
5) *Every σ-discrete open covering has a finite covering.*

Proof. 2)$\rightarrow$1), 3)$\rightarrow$1) and 4)$\rightarrow$1) are obvious by Theorem 3 in [4]. 5)$\rightarrow$1) follows from the definition of countably compactness.

We must prove that (1) implies the other propositions (2), (3), (4) and (5). In general, (2)$\rightarrow$(3), (3)$\rightarrow$(4) are trivial.

First, we shall show (1)$\rightarrow$(2). Let α be a σ-point finite open covering of S, then there is a system of family α_n of open sets such that $\alpha=\bigcup_i\alpha_i$ and each α_i is point finite. Let O_n be the sum of all members of α_n, then $\beta=\{O_n\}$ is a countable open covering of S. Since S is countably compact, β has a finite covering $\{O_{n_1},\cdots,O_{n_k}\}$. Then the system $\gamma=\{\alpha_{n_1},\cdots,\alpha_{n_k}\}$ is an open covering and it is obvious that γ is a point finite covering. Therefore, by Theorem 2 in [3], γ has a finite covering. This shows (1)$\rightarrow$(2). Second, we shall show