51. On Generating Elements of Simple Rings

By Hisao TOMINAGA and Friedrich KASCH
Mathematical Institutes, Okayama University and Heidelberg University
(Comm. by Z. SUETUNA, M.J.A., April 12, 1957)

1. **Introduction.** Some years ago, one of the present authors investigated in what range the classical theorem for the existence of a primitive element is carried over to Galois extensions of division rings, and proved the following: Any Galois extension K of a division subring H of finite rank possesses two generating elements over H [4, Satz 14]. In particular, if the center of the centralizer of H in K is separable over the center of K then there exist two generating elements of K/H which are conjugate (with respect to an inner automorphism) [2, Satz 5].

Regarding the corresponding question in ring extensions the author also sharpened Albert's result in [1] as follows: In any separable algebra A/F, where F is an infinite field, there exist two generating elements over F which are conjugate [3].

Recently these problems are reexamined: T. Nagahara succeeded first in excluding the separability hypothesis assumed in [2, Satz 5] [5, Theorem 4], and by the light of this fact, another of the present authors showed that any simple ring which is Galois and finite over a simple subring S is generated over S by three regular elements [7, Theorem 2].

In what follows, we shall prove that Nagahara's result is still valid for simple rings (Theorem 1), and that the result in [3] restricted to simple algebras is true without the infiniteness hypothesis for the ground field (Theorem 2).

2. **Fundamental lemma.** By a *simple ring* we shall mean, throughout this note, a two-sided simple ring with an identity satisfying minimum condition for one-sided ideals. And we say a simple ring R is *Galois* over a simple subring S if S is the fixed subring of an automorphism group in R and $V_R(S)$ (the centralizer of S in R) is simple. At last, for any subring U and any subset X in a ring T, $U(X)$ will mean the subring of T generated by X over U.

Let a ring T with an identity be finite over a simple subring $U=\sum_1^m Kc_{ij}$ containing the identity of T as a U-left module, where c_{ij}'s are matric units and $K=V_U(\{c_{ij}\})$ is a division ring. As evidently T is finite over K, we can easily see that, *for any regular element* $t\in T$, t^{-1} *is contained in* $K(t)$, *so that in* $U(t)$. This fact will be used very often in the sequel.