

69. A Note on Compact Space

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In this short Note, we shall give a remark for my results [2, 4]. Following E. Hewitt, we shall define a pseudo-compact space as follows. A completely regular T_2 -space S is called pseudo-compact, if every continuous function on S is bounded. In his paper [1], R. Doss has proved that if, in a uniform T_2 -space S , every continuous function reaches its upper bound, then S is precompact. This is a generalisation of a result of M. Fréchet. Therefore, this implies the following result. Any pseudo-compact uniform space is precompact. Hence, any pseudo-compact complete uniform space is compact. It is well known that every uniform space with Lebesgue property is paracompact complete (see [3] or [5]). Hence, by Theorem 2 in [4], we have the following propositions:

Proposition 1. Every pseudo-compact complete uniform space is compact.

Proposition 2. The following statements of a uniform space S with Lebesgue property are equivalent:

- 1) S is compact.
- 2) S is pseudo-compact.
- 3) S is countably compact.

Proposition 2 was obtained in [2] by an other method.

References

- [1] R. Doss: On continuous functions in uniform spaces, *Ann. Math.*, **48**, 843-844 (1947).
- [2] K. Iséki: On the property of Lebesgue in uniform spaces. III, *Proc. Japan Acad.*, **31**, 441-442 (1955).
- [3] K. Iséki: On the Lebesgue property in uniform spaces, *Publ. Math., Debrecen*, **4**, 239-241 (1956).
- [4] K. Iséki: A remark on countably compact normal space, *Proc. Japan Acad.*, **33**, 131-133 (1957).
- [5] S. Kasahara: On the Lebesgue property in uniform spaces, *Math. Japonicae*, **3**, 127-132 (1955).